

國立台灣科技大學一百學年度碩士班招生試題

系所組別：材料科學與工程學系碩士班乙組

科目：工程數學

(總分為100分)

總分 100 分，共 7 大題。

1. At each point (x,y) of a curve the intercept of the tangent on the y -axis is equal to $2xy^2$. Find the curve. (10%)

2. Obtain the response of the following system: (10%)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

when the input u is the unit-step function. Assume that $x_1(0) = 0, x_2(0) = 0$.

3. Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2) \frac{dy}{dx} + y = 3x+4$ (10%)

4. Find the eigenvalues and the eigenfunction corresponding to each eigenvalue for the following Sturm-Liouville problems.

(a) $y'' + \lambda y = 0; \quad y(0) = y(\pi), \quad y'(0) = y'(\pi)$ (10%)

(b) $y'' - 12y' + 4(7 + \lambda)y = 0; \quad y(0) = y(5) = 0$ (10%)

5. Solve the initial value problem: (15%)

$$y'' + 5y' + 6y = 3\delta(t-2) + f(t); \quad y(0) = 0, \quad y'(0) = 4, \text{ with}$$

$$f(t) = \begin{cases} 0 & 0 \leq t < 3\pi/2 \\ 10 \cos t & t \geq 3\pi/2 \end{cases}$$

6. Solve the differential equation. (15%)

$$y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}$$

$$y(0) = 0.5, y'(0) = 2.5, y''(0) = -4.5$$

7. Find the inverse transform of given functions.

a) $\cos t \mathcal{Z}'(t-\pi)$ (10%)

b) $\frac{1}{(s^2 + k^2)^2}$ (10%)

