

## 國立臺灣科技大學101學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組  
科 目： 工程數學

(總分為100分)

1. Solve the following ordinary differential equations:

$$(1) y' - xy^2 - (1 - 2x)y - x = -1 \quad (10\%)$$

$$(2) y'' - y' - 12y = 2\sinh^2(x) \quad (10\%)$$

2. Use Laplace Transform method to solve the following equation. (15%)

$$y' + 5y + 6 \int_0^t y dt = 8H(t - 4)$$

$$\text{where } y(0) = 2 \quad \text{and } H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

3. Consider a 3x3 matrix A.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

(1) Find the eigenvalues and the corresponding eigenvectors. (10%)

(2) Find a matrix P to diagonalize the matrix A. (5%)



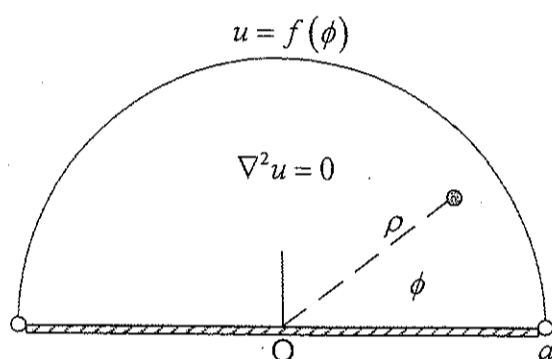
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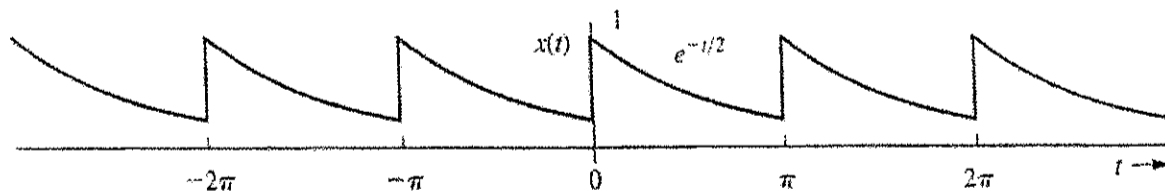
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4. Derive an expression for the steady temperatures  $u = u(\rho, \phi)$  in a long rod, with a uniform semicircular cross section and occupying the region  $0 \leq \rho \leq a$ ,  $0 \leq \phi \leq \pi$ , which is insulated on its planar surface and maintained at temperatures  $f(\phi)$  on the semicircular part in Problem 4.  $u(\rho, \phi)$  satisfies Laplace's equation  $\rho^2 u_{\rho\rho}(\rho, \phi) + \rho u_{\rho}(\rho, \phi) + u_{\phi\phi}(\rho, \phi) = 0$ . (15%)



Problem 4

5. The compact form of trigonometric Fourier series for a periodic signal  $x(t)$  is  $C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$ . With the use of such a compact representation, obtain the trigonometric Fourier series for the signal  $e^{-t/2}$  shown in Problem 5. Sketch the amplitude and phase with respect to  $n\omega_0$  for  $e^{-t/2}$ . (20%)



Problem 5

6. Let  $f(z) = \frac{2i}{4+iz}$ . Find the Taylor expansion of  $f(z)$  about  $-3i$  and specify the radius of convergence. (15%)

