國立臺灣科技大學101學年度碩士班招生試題

系所組別： 資訊工程系碩士班

科　　目： 計算機數學

(總分為100分)

1. (8%) 90% of new airport-security personnel have had prior training in weapon detection. During their first month on the job, personnel without prior training fail to detect a weapon 3% of the time, while those with prior training fail only 0.5% of the time. What is the probability a new airport-security employee, who fails to detect a weapon during the first month on the job, has had prior training in weapon detection?

2. (7%) For all $n \in Z$, $n \geq 0$, prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

3. (15%) Let an alphabet $\sum = \{0, 1\}$ and a language $A = \{0, 01, 011, 0111, 1111\} \subseteq \sum^*$. For $n \geq 1$, let a_n count the number of strings in the *Kleene closure* A^* of length n.

   (a) Find a recurrence relation for a_n. (7%)

   (b) Solve the recurrence relation in (a). (8%)

4. (10%)

   (a) Let $T = (V, E)$ be a tree with $|V| = n \geq 2$. How many distinct paths are there in the tree, T? (5%)

   (b) A tree has three vertices of degree 2, two vertices of degree 3, and four vertices of degree 4. How many vertices of degree 1 does it have?(5%)

5. (10%) In how many ways can 16 different books be distributed among four children so that

   (a) each child gets four books? (5%)

   (b) the two oldest children get five books and the two youngest get three books each? (5%)

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6. (10%) Suppose that $A = S\Lambda S^{-1}$, where Λ is a diagonal matrix with diagonal elements $\lambda_1, \lambda_2, \ldots, \lambda_n$ and S is an $n \times n$ matrix with columns $\mathbf{s}_i$, for $i = 1, 2, \ldots, n$.

   (a) Show that if $\mathbf{x} = \sum_{i=1}^{n} \alpha_i \mathbf{s}_i$, then $A^k\mathbf{x} = \sum_{i=1}^{n} \alpha_i \lambda_i^k \mathbf{s}_i$, where k is a positive integer. (5%)

   (b) Suppose that $|\lambda_i| < 1$ for $i = 1, 2, \ldots, n$. What happens to $A^k\mathbf{x}$ as $k \to \infty$? Explain. (5%)

7. (10%) The linear transformation T is defined by $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$$

   Find the kernel of T.

8. (15%) Determine whether each statement is *true* or *false*. If the statement is *true*, prove it. Otherwise provide a counterexample.

   (a) Let A and B be $n \times n$ matrices and A is similar to B. If A is a nonsingular matrix then B is nonsingular too. (5%)

   (b) Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a set of linearly dependent vectors in V, and let T be a linear transformation from V to V. Then the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_n)\}$ is linearly dependent. (5%)

   (c) Let A be an $n \times n$ matrix. If the rank of A is strictly less than n, then the system of linear equations $Ax = b$ has infinite many solutions for any vector $b \in R^n$. (5%)

9. (15%) If the $n \times n$ matrix A can be written as the product of a lower triangular matrix L and an upper triangular matrix U, then $A = LU$ is an LU-factorization of A.

   (a) Find the LU-factorization of A, where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ -1 & -3 & 0 \end{bmatrix}. \text{ (8\%)}$$

   (b) Let A be the matrix given in (a) and $b = [-1, 0, 2]^\top$. Solve the system of equations $A\mathbf{x} = b$ by the LU-factorization result from (a). (7%)