

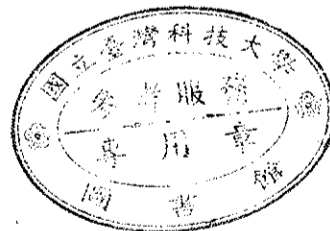
國立臺灣科技大學101學年度碩士班招生試題

系所組別：資訊工程系碩士班

科目：計算機數學

(總分為100分)

1. (8%) 90% of new airport-security personnel have had prior training in weapon detection. During their first month on the job, personnel without prior training fail to detect a weapon 3% of the time, while those with prior training fail only 0.5% of the time. What is the probability a new airport-security employee, who fails to detect a weapon during the first month on the job, has had prior training in weapon detection?
2. (7%) For all $n \in \mathbb{Z}$, $n \geq 0$, prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.
3. (15%) Let an alphabet $\Sigma = \{0, 1\}$ and a language $A = \{0, 01, 011, 0111, 1111\} \subseteq \Sigma^*$. For $n \geq 1$, let a_n count the number of strings in the Kleene closure A^* of length n .
 - (a) Find a recurrence relation for a_n . (7%)
 - (b) Solve the recurrence relation in (a). (8%)
4. (10%)
 - (a) Let $T = (V, E)$ be a tree with $|V| = n \geq 2$. How many distinct paths are there in the tree, T ? (5%)
 - (b) A tree has three vertices of degree 2, two vertices of degree 3, and four vertices of degree 4. How many vertices of degree 1 does it have? (5%)
5. (10%) In how many ways can 16 different books be distributed among four children so that
 - (a) each child gets four books? (5%)
 - (b) the two oldest children get five books and the two youngest get three books each? (5%)



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6. (10%) Suppose that $A = SAS^{-1}$, where Λ is a diagonal matrix with diagonal elements $\lambda_1, \lambda_2, \dots, \lambda_n$ and S is an $n \times n$ matrix with columns s_i , for $i = 1, 2, \dots, n$.

(a) Show that if $x = \sum_{i=1}^n \alpha_i s_i$, then $A^k x = \sum_{i=1}^n \alpha_i \lambda_i^k s_i$, where k is a positive integer. (5%)

(b) Suppose that $|\lambda_i| < 1$ for $i = 1, 2, \dots, n$. What happens to $A^k x$ as $k \rightarrow \infty$? Explain. (5%)

7. (10%) The linear transformation T is defined by $T(x) = Ax$ where

$$A = \begin{bmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$$

Find the kernel of T .

8. (15%) Determine whether each statement is *true* or *false*. If the statement is *true*, prove it. Otherwise provide a counterexample.

(a) Let A and B be $n \times n$ matrices and A is similar to B . If A is a nonsingular matrix then B is nonsingular too. (5%)

(b) Let $S = \{v_1, v_2, \dots, v_n\}$ be a set of linearly dependent vectors in V , and let T be a linear transformation from V to V . Then the set $\{T(v_1), T(v_2), \dots, T(v_n)\}$ is linearly dependent. (5%)

(c) Let A be an $n \times n$ matrix. If the rank of A is strictly less than n , then the system of linear equations $Ax = b$ has infinite many solutions for any vector $b \in R^n$. (5%)

9. (15%) If the $n \times n$ matrix A can be written as the product of a lower triangular matrix L and an upper triangular matrix U , then $A = LU$ is an LU-factorization of A .

(a) Find the LU-factorization of A , where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ -1 & -3 & 0 \end{bmatrix}. \quad (8\%)$$

(b) Let A be the matrix given in (a) and $b = [-1, 0, 2]^T$. Solve the system of equations $Ax = b$ by the LU-factorization result from (a). (7%)

