國立臺灣科技大學101學年度碩士班招生試題

系所組別： 工業管理系碩士班甲組、乙組、丙組

科　　目： 統計學

（總分為100分）

(Total 100 Points.) ***There are 6 Problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.***

1. (10 points) The life of a DVD player has an exponential distribution with an average life $\beta = 3$ years. If 150 of these DVD players are purchased for different houses, what is the probability that at most 50 fail during the first year?

2. (20 points) Random variables X and Y has the following joint probability distribution:

$$f(x,y)=\begin{cases} k(\frac{1}{2}xy+y^2), & 0<x<2, 0<y<1, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find k. (5 points)
   (b) Find marginal probability distribution of the random variable X. (5 points)
   (c) Find conditional probability distribution of Y given that $X = 2$. (5 points)
   (d) Find $E[E(y \mid x)]$. (5 points)

3. (20 points) The beta distribution has considerable application in reliability problems in which the basic random variable is a proportion. If the proportion of a DVD player requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, and the density function is given by

$$f(x;\alpha,\beta)=\begin{cases} \dfrac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}, & 0<x<1, \\ 0, & \text{otherwise.} \end{cases}$$

   where $B(\alpha, \beta)$ is the beta function defined by

$$B(\alpha,\beta)=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)},$$ for $\alpha, \beta > 0$, and $\Gamma(\alpha)$ is the gamma function.

   (a) What is the probability that at least 70% of the new models of this brand sold this year will require service during their first year of operation? (8 points)
   (b) What is the mean proportion of a DVD player requiring service during the first year of operation? (6 points)
   (c) What is the variance of this random variable? (6 points)

4. (16 points) Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specially, catalyst 1 is currently in use, but catalyst 2 is acceptable. Since catalyst 2 is cheaper, it should be adopted, providing it does not change the process yield. An experiment is run in the pilot plant and results in the data shown in the following table. Assume that the measurements are normally distributed.

Obs. No.	1	2	3	4	5	6	7	8	$\bar{x}$	s
Catalyst 1	89.2	94.7	89.1	91.8	95.4	92.2	94.2	91.5	92.26	2.38
Catalyst 2	92.8	91.1	97.0	97.2	93.2	90.5	91.0	89.2	92.75	2.96

國立臺灣科技大學101學年度碩士班招生試題

系所組別： 工業管理系碩士班甲組、乙組、丙組

科　　目： 統計學

（總分為100分）

(a) Construct a 95% confidence interval on the ratio σ_1^2 / σ_2^2. (8 points)

(b) Based on the results in part (a), is there any difference between the mean yields for two types of catalysts? Use $\alpha = 0.05$. (8 points)

5. (24 points) A manufacturer of television sets is interested in the effect of tube conductivity of four different types of coating for color picture tubes. A completely randomized experiment is conducted and the following conductivity data are obtained:

Coating Type	Conductivity			
1	143	141	150	146
2	152	149	138	143
3	129	127	132	130

(a) Is there a difference in mean conductivity due to coating type? Set up the appropriate hypothesis for investigating this issue. Use $\alpha = 0.05$. (12 points)

(b) Estimate the overall mean and the treatment effects. (8 points)

(c) If wish to minimize conductivity, which type of coating for color picture tubes would you recommend to the manufacturer? (4 points)

6. (10 points) Three cards are drawn from an ordinary deck of playing cards, with replacement, and the number of spades (Y) is recorded. After repeating the experiment 64 times, the following outcomes were recorded:

y	0	1	2	3
f	20	32	11	1

Test the hypothesis of 0.05 level of significance that the recorded data may be fitted by the binomial distribution $b(y;3,1/4)$, $y = 0,1,2,3$.

501101、501201、501301

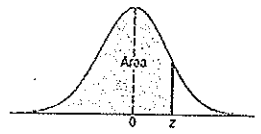
國立臺灣科技大學101學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科　　目：統計學

（總分為100分）

Areas under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

國立臺灣科技大學101學年度碩士班招生試題

系所組別： 工業管理系碩士班甲組、乙組、丙組

科　　目： 統計學

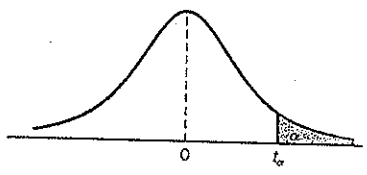
（總分為100分）

Note: F-distribution with degrees of freedom v_1 and v_2

$$P\left(F_{v1,v2} > f_{0.05}(v_1, v_2)\right) = 0.05$$

	$f_{0.05}(v_1, v_2)$								
	v_1								
v_2	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59

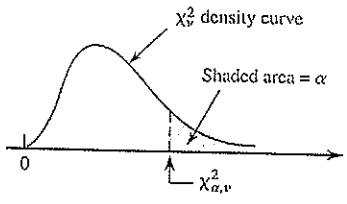
Note: t distribution with degrees of freedom v



$$P\left(T_v > t_{\alpha,v}\right) = \alpha$$

	α			α	
v	0.05	0.025	v	0.05	0.025
1	6.314	12.706	16	1.746	2.120
2	2.920	4.303	17	1.740	2.110
3	2.353	3.182	18	1.734	2.101
4	2.132	2.776	19	1.729	2.093
5	2.015	2.571	20	1.725	2.086
6	1.943	2.447	21	1.721	2.080
7	1.895	2.365	22	1.717	2.074
8	1.860	2.306	23	1.714	2.069
9	1.833	2.262	24	1.711	2.064
10	1.812	2.228	25	1.708	2.060
11	1.796	2.201	26	1.706	2.056
12	1.782	2.179	27	1.703	2.052
13	1.771	2.160	28	1.701	2.048
14	1.761	2.145	29	1.699	2.045
15	1.753	2.131	30	1.697	2.042

Note: χ^2_v distribution with degrees of freedom v



$$P\left(\chi^2_v > \chi^2_{\alpha,v}\right) = \alpha$$

Table Critical Values of the Chi-Squared Distribution

	α			
v	0.99	0.975	0.95	0.90
1	0.0^3157	0.0^3982	0.00393	0.0158
2	0.0201	0.0506	0.103	0.211
3	0.115	0.216	0.352	0.584
4	0.297	0.484	0.711	1.064
5	0.554	0.831	1.145	1.610
6	0.872	1.237	1.635	2.204
7	1.239	1.690	2.167	2.833
8	1.647	2.180	2.733	3.490

	α			
v	0.10	0.05	0.025	0.01
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.832	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090