

國立臺灣科技大學101學年度碩士班招生試題

系所組別：工業管理系碩士班甲組

科目：作業研究

(總分為100分)

1. Consider the following linear program:

$$\max Z = 2x_1 + 3x_2$$

s.t.

$$5x_1 + 2x_2 \leq 8$$

$$7x_1 + 3x_2 \leq 10$$

$$x_1, x_2 \geq 0$$

- (a) Solve this linear program by the revised simplex method. (10%)
 (b) Find the dual of this linear program and its optimal solution. (10%)
 (c) Find the range of values of b_2 , i.e. the right hand side of the 2nd constraint, for which the optimal basis obtained in part (a) remains optimal. (5%)
2. A company sells 8 types of boxes, ranging in volume from 15 to 50 cubic feet. The demand and size of each box is given in the following table.

	Box							
	1	2	3	4	5	6	7	8
Volume	50	45	40	35	30	25	20	15
Demand	400	300	700	500	300	600	400	200

The variable cost (in dollars) of producing each box is equal to the box's volume. A fixed cost of \$1,200 is incurred to produce any of a particular box. If the company desires, demand for a box may be satisfied by a box of larger size.

- (a) Formulate a shortest path problem whose solution will minimize the cost of meeting the demand for boxes. (10%)
 (b) Solve the shortest path problem by the Dijkstra's algorithm. (5%)
3. There is an object whose weight is w kg. There are n types of stones, each stone of type i weighs exactly a_i kg., for $i = 1$ to n ; and an unlimited number of copies of each are available. w, a_1, \dots, a_n are given positive integers. The object is placed in the right pan of a balance. It is required to place stones in the right and/or left pans of the balance so that it becomes perfectly balanced. It is required to do this using the smallest possible number of stones. Formulate this problem as an integer program. (10%)



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4. Customers arrive at a two-server service station according to a Poisson process with rate λ . Whenever a new customer arrives, any customer that is in the system immediately departs. A new arrival enters service first with server 1 and then with server 2. If the service times at the servers are independent exponentials with respective rates μ_1 and μ_2 , what proportion of entering customers completes their service with server 2? (15 %)
5. Each time a machine is repaired it remains up for an exponentially distributed time with rate λ . It then fails, and its failure is either of two types. If it is a type 1 failure, then the time to repair the machine is exponential with rate μ_1 ; if it is a type 2 failure, then the time to repair the machine is exponential with rate μ_2 . Each failure is, independently of the time it took the machine to fail, a type 1 failure with probability p and a type 2 failure with probability $1-p$. What proportion of time is the machine down due to a type 1 failure? What proportion of time is it down due to a type 2 failure? What proportion of time is it up? (20 %)
6. An advertising firm has D dollars to spend on reaching customers in T separate markets. Market t consists of k_t people. If x dollars are spent on advertising in market t , the probability that a given person in market t will be reached is $p_t(x)$. Each person in market t who is reached will buy c_t units of the product. A person who is not reached will not buy any of the product. Formulate a dynamic programming recursion that could be used to maximize the expected number of units sold in T markets. (15 %)

