

## 國立臺灣科技大學102學年度碩士班招生試題

系所組別：化學工程系碩士班

科目：工程數學與輸送現象

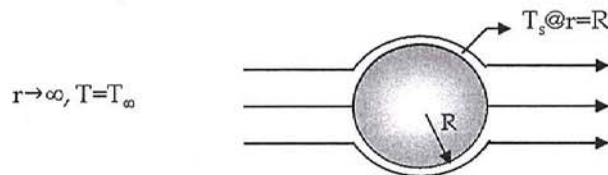
(總分為100分)

1. (a) For a three-dimensional irrotational flow, please prove that the velocity can be expressed as  $\vec{v} = \nabla\phi$ , where  $\phi$  is velocity potential. (10%)
- (b) Please present the equation of motion (Navier-Stokes equation) as dimensionless form with Strouhal number, **Ruark** number and **Reynold** number.

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\nabla P + \nu \nabla^2 \vec{v}$$

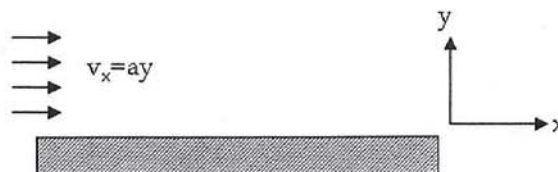
where  $St = \frac{v_0}{L_0/t_0}$ ,  $Ru = \frac{\rho v_0}{\rho_0}$ ,  $Re = \frac{\rho v_0 L_0}{\mu}$  and  $v_0$ ,  $L_0$ ,  $t_0$  and  $\rho_0$  are characteristic velocity, length, time and pressure, respectively. (10%)

2. Consider a heated sphere is suspended in a large, motionless surrounding fluid. Show that **Nusselt** number=2 for heat transfer from sphere at low **Reynold** number.



where  $\nabla = \frac{\partial}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \hat{e}_\phi$ , and  $\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$  (20%)

3. A liquid flows over a thin and flat sheet of slightly soluble solid. Over the region in which *diffusions along x and y direction* are occurring, the liquid velocity may be assumed to be parallel to the plate and to be given by  $v_x = ay$ , where  $y$  is the vertical distance from the plate and  $a$  is a constant. Show that the equation of mass transfer at steady-state. (10%)



4. Solve the given initial-value problem?

(a)  $y'' + 4y' + 13y = \delta(t - \pi) + \delta(t - 2\pi)$ ,  $y(0) = 1$ ,  $y'(0) = 0$  (10%)

(b)  $\frac{dy}{dt} + 6y(t) + 9 \int_0^t y(\tau) d\tau = 1$ ,  $y(0) = 0$  (10%)



## 國立臺灣科技大學102學年度碩士班招生試題

系所組別： 化學工程系碩士班

科 目： 工程數學與輸送現象

(總分為100分)

5. Solve the boundary value problem using separation of variables. (20%)

$$k \frac{\partial^2 u}{\partial x^2} + \sin 2\pi x = \frac{\partial u}{\partial t}, 0 < x < 1, t > 0$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = \sin \pi x$$

6. Compute the work done by  $F(x, y) = (e^x - y)\mathbf{i} + (e^y \cos y + x)\mathbf{j}$  in moving a particle counterclockwise about the rectangular path C having vertices (0,0), (2,0), (2,4) and (0,4). (10%)

