國立臺灣科技大學102學年度碩士班招生試題

系所組別： 資訊工程系碩士班

科　　目： 計算機數學

(總分為100分)

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1. [12%] Suppose that p_1, p_2, p_3 are distinct primes and that n, $k \in Z^+$ with $n = p_1^5 p_2^3 p_3^k$. Let A be the set of positive integer divisors of n and define the relation $\mathcal{R}$ on A by $x\mathcal{R}y$ if x exactly divides y. If there are 5880 ordered pairs in $\mathcal{R}$, determine k and $|A|$.

2. [12%] Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if
   (4%) (a) a 0 bit and a 1 bit are equally likely
   (4%) (b) the probability that a bit is a 1 is 0.6
   (4%) (c) the probability that the ith bit is a 1 is $1/2^i$ for $i = 1, 2, 3, \ldots, 10$

3. [13%] A computer dating service wants to match each of four women with one of six men. According to the information these applicants provided when they joined the service, we can draw the following conclusions.
   - Woman 1 would not be compatible with man 1, 3, or 6.
   - Woman 2 would not be compatible with man 2 or 4.
   - Woman 3 would not be compatible with man 3 or 6.
   - Woman 4 would not be compatible with man 4 or 5.

   In how many ways can the service successfully match each of the four women with a compatible partner?

4. [13%] How many different channels are needed for six stations located at the distances shown in the following table, if two stations cannot use the same channel when they are within 150 miles of each other?

Station	1	2	3	4	5	6
1	—	85	175	200	50	100
2	85	—	125	175	100	160
3	175	125	—	100	200	250
4	200	175	100	—	210	220
5	50	100	200	210	—	100
6	100	160	250	220	100	—

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5. [5%] If $f: \mathbb{R}^n \to \mathbb{R}$ is a linear function, given $\alpha \in \mathbb{R}^n$ and $f(\alpha) \neq 0$, is $f_1 = f + f(\alpha)$ also a linear function? Prove your answer.

6. [15%] In $\mathbb{R}^3$, let $\alpha_1 = (1,0,1)$, $\alpha_2 = (0,1,-2)$, $\alpha_3 = (-1,-1,0)$. If f is a linear function from $\mathbb{R}^3$ to $\mathbb{R}$ such that

   $f(\alpha_1) = 1,\ f(\alpha_2) = -1,\ f(\alpha_3) = 3.$

   (10%) (a) Can we find $f((a,b,c))$ for any given $\alpha = (a,b,c)$? Write down $f((a,b,c))$ if your answer is "yes" or simply say "no" followed by your reason.

   (5%) (b) What is the dimensionality of the vector space V formed by all linear functions $f: \mathbb{R}^3 \to \mathbb{R}$, similar to the above case? Explain your answer and give a basis in the space V.

   [Hint: one element in the basis can be a function f_1 where we have $f_1(\alpha_1) = 1$, $f_1(\alpha_2) = 0$, $f_1(\alpha_3) = 0$. What others?]

7. [20%]

   (10%) (a) Let A and B be $n \times n$ matrices satisfying $A\mathrm{x} = B\mathrm{x}$ for all $\mathrm{x} \in \mathbb{R}^n$. Find the *null space* and *nullity* of $A - B$.

   (5%) (b) Find the *row rank* of $[A - B \quad C]$ where C is a $n \times m$ matrix and has *column rank* k.

   (5%) (c) Let A and B be $n \times n$ matrices. If the matrix $(I - AB)$ is invertible, will the matrix $(I - BA)$ also be invertible? If your answer is "yes", find the inverse of $(I - BA)$ in term of the inverse of $(I - AB)$; i.e., represent

   $$(I - BA)^{-1} = f((I - AB)^{-1})$$

   for some function f. On the other hand, if your answer is "no", prove your answer. [Hint: think of $(I - X)^{-1}$ as $\sum_{i=0}^{\infty} X^i$]

8. [10%] Let $A = \begin{bmatrix} -4 & 15 \\ -2 & 7 \end{bmatrix}$, find the general form of $A^{-k} = (A^{-1})^k$ for some positive integer k.