

## 國立臺灣科技大學102學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科目：統計學

(總分為100分)

**(Total 100 Points)** *There are 6 Problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.*

1. (10 points) A random variable  $X$  has the following probability distribution, where  $c$  is a constant:

$$f(x) = \begin{cases} \frac{1}{2} x^2 e^{-cx}, & 0 \leq x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

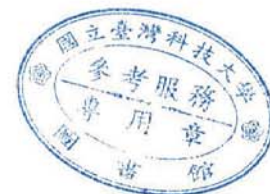
- (a) Find the value of the constant  $c$ . (5 points)  
 (b) Find the probability distribution of  $Y$ , where  $Y = X^2$ . (5 points)
2. (20 points) The beta distribution has considerable application in reliability problems in which the basic random variable is a proportion. Suppose the random variable  $X$  follows a beta distribution with  $\alpha = 1$  and  $\beta = 3$ , and the density function is given by

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

where  $B(\alpha, \beta)$  is the beta function defined by

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \text{ for } \alpha, \beta > 0, \text{ and } \Gamma(\alpha) \text{ is the gamma function.}$$

- (a) Determine the mean of  $X$ . (5 points)  
 (b) Determine the median of  $X$ . (5 points)  
 (c) Determine the variance of  $X$ . (5 points)  
 (d) Find the probability that  $X > \frac{1}{2}$ . (5 points)
3. (20 points) Two continuous random variables  $X_1$  and  $X_2$  are independent. Suppose that  $X_1$  and  $X_2$  are exponential distributions with parameters  $\beta_1 = 1$  and  $\beta_2 = 1$ , respectively. Moreover, let the random variables  $Y_1 = X_1$  and  $Y_2 = X_1 + X_2$ .
- (a) Find the probability distributions of  $X_1$  and  $X_2$ , respectively. (5 points)  
 (b) Find the joint probability distribution of  $X_1$  and  $X_2$ . (5 points)  
 (c) Find the joint probability distribution of  $Y_1$  and  $Y_2$ . (5 points)  
 (d) Find the marginal probability distribution of  $Y_2$ . (5 points)



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4. (15 points) Advertisements by NTUST claim that their new drug can effectively decrease cholesterol levels over a six-week period. A random sample of 9 recent participants showed the following cholesterol levels measured before and after the six-week treatment period. Suppose we want to test the claim in the advertisement.

Participant	1	2	3	4	5	6	7	8	9
Before	300	250	250	220	220	180	180	150	140
After	295	240	225	210	210	170	185	140	125

- (a) State the null hypothesis and the alternative hypothesis, conduct appropriate test, and draw appropriate conclusions (at the 0.05 significance level) (10 points)
- (b) What assumption needs to be made about the distribution of the differences? (2 points)
- (c) Why matching or pairing samples is better than independent samples here? (3 points)
5. (25 points) For a simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \quad i = 1, 2, \dots, n,$$

assumed that  $E(\epsilon_i) = 0$ ,  $\text{Var}(\epsilon_i) = \sigma^2$ , and the errors  $\epsilon_1, \epsilon_2, \dots, \epsilon_n$  are independent.

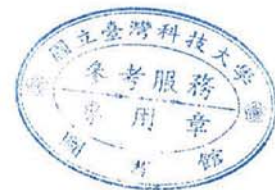
- (a) Please shows that the mean squared error (MSE) is an unbiased estimate of  $\sigma^2$  (10 points)

Now let  $Y_i$  be the price for 6 randomly selected workstation computers and  $x_i$  be their corresponding processor speeds in gigahertz:

speed (gigahertz)	1	1	2	2	3	3
price (\$1000s)	28	32	36	44	42	58

- (b) Find an estimated regression equation  $\hat{Y} = b_0 + b_1 X$  that can be used to predict price given the speed of processor. (5 points)
- (c) Test the relationship between speed of processor and price? What's your conclusion? Use  $\alpha = 0.05$ . (5 points)
- (d) Prepare a residual plot of  $Y - \hat{Y}$  versus  $X$ . What conclusion can you draw from this residual analysis? What may you do to fix this problem? (5 points)
6. (10 points) The manufacturing facility has collected and summarized its data and calculated the number of defectives for four different machines. The manager want to determine whether or not the proportion of defectives produced by the four machines was the same. Please state the null and alternative hypothesis, perform the test, and state your conclusion with a 0.05 level of significance.

Number of	Machine			
	A	B	C	D
Defective	12	22	15	11
Good	188	178	185	189



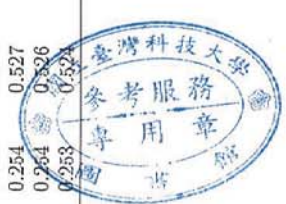
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Table 1. Critical Values of the *t*-Distribution

<i>v</i>	$\alpha$														
	0.40	0.30	0.20	0.15	0.10	0.05	0.025	<i>v</i>	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
$\infty$	0.253	0.524	0.842	1.036	1.282	1.645	1.960	$\infty$	2.054	2.170	2.326	2.432	2.576	2.807	3.290



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Table 2. Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.004303	0.0157	0.02628	0.03852	0.05405	0.07579	0.10645	0.14868	0.20154	0.33707
2	0.010001	0.02011	0.03000	0.04000	0.05000	0.06446	0.08328	0.10645	0.13579	0.21478
3	0.07717	0.1115	0.1385	0.1668	0.2001	0.2398	0.2859	0.3398	0.4013	0.58409
4	0.207	0.297	0.392	0.484	0.577	0.675	0.777	0.882	0.990	1.38482
5	0.412	0.554	0.702	0.831	0.975	1.124	1.284	1.455	1.637	2.20413
6	0.676	0.872	1.134	1.237	1.395	1.564	1.744	1.935	2.149	2.83307
7	0.989	1.239	1.564	1.690	1.833	2.002	2.182	2.373	2.575	3.35425
8	1.344	1.647	2.032	2.180	2.333	2.502	2.682	2.873	3.074	3.85841
9	1.735	2.088	2.532	2.700	2.825	3.005	3.195	3.395	3.595	4.34665
10	2.156	2.558	3.059	3.247	3.390	3.580	3.770	3.970	4.170	4.81517
11	2.603	3.053	3.609	3.816	3.975	4.165	4.365	4.575	4.785	5.27202
12	3.074	3.571	4.178	4.404	4.526	4.716	4.916	5.126	5.336	5.71538
13	3.565	4.107	4.765	5.009	5.092	5.282	5.482	5.682	5.882	6.14459
14	4.075	4.660	5.368	5.629	5.671	5.861	6.061	6.261	6.461	6.55073
15	4.601	5.229	5.985	6.262	6.261	6.451	6.651	6.851	7.051	6.93786
16	5.142	5.812	6.614	6.908	6.862	7.052	7.252	7.452	7.652	7.31436
17	5.697	6.408	7.255	7.564	7.526	7.716	7.916	8.116	8.316	7.68136
18	6.265	7.015	7.906	8.231	8.190	8.380	8.580	8.780	8.980	8.03936
19	6.844	7.633	8.567	8.907	8.861	9.051	9.251	9.451	9.651	8.38836
20	7.434	8.260	9.237	9.591	9.541	9.731	9.931	10.131	10.331	8.72836
21	8.034	8.897	9.915	10.283	10.228	10.418	10.618	10.818	11.018	9.05936
22	8.643	9.542	10.600	10.982	10.922	11.112	11.312	11.512	11.712	9.38136
23	9.260	10.196	11.293	11.689	11.622	11.812	12.012	12.212	12.412	9.69436
24	9.886	10.856	11.992	12.401	12.328	12.518	12.718	12.918	13.118	10.00736
25	10.520	11.524	12.697	13.120	13.042	13.232	13.432	13.632	13.832	10.31136
26	11.160	12.198	13.409	13.844	13.761	13.951	14.151	14.351	14.551	10.61536
27	11.808	12.878	14.125	14.573	14.485	14.675	14.875	15.075	15.275	10.91936
28	12.461	13.565	14.847	15.308	15.215	15.405	15.605	15.805	16.005	11.22336
29	13.121	14.256	15.574	16.047	15.949	16.139	16.339	16.539	16.739	11.52736
30	13.787	14.953	16.306	16.791	16.691	16.881	17.081	17.281	17.481	11.83136
40	20.707	22.164	23.898	24.433	24.309	24.499	24.699	24.899	25.099	12.70136
50	27.991	29.707	31.664	32.357	32.164	32.354	32.554	32.754	32.954	13.44136
60	35.534	37.485	39.699	40.482	40.188	40.377	40.577	40.777	40.977	14.18136

