

國立臺灣科技大學102學年度碩士班招生試題

系所組別：材料科學與工程系碩士班乙組

科目：工程數學

(總分為100分)

總分 100 分，共 8 大題。

1. Find the Laplace transform of $f(t) = 2e^{-t} \cos 10t - t^4 + 6e^{-(t-10)}$ for $t > 0$. (10%)

2. Find the inverse Laplace transform of $F(s) = \frac{2e^{-0.5s}}{s^2 - 6s + 13} - \frac{s-1}{s^2 - 2s + 2}$ for $t > 0$. (10%)

3. Solve $y(xy+1)dx + x(1+xy+x^2y^2)dy = 0$. (10%)

4. Solve the initial value problem (10%)

$$x_1' = x_1 - 4x_2,$$

$$x_2' = 2x_1 - 5x_2; \quad x_1(0) = 1, \quad x_2(0) = -1$$

5. A parachutist is falling with speed 176 ft/sec when his parachute opens. If the air resistance is $\frac{Wv^2}{256}$ lb, where W is the total weight of the man and parachute, find his speed v as a function of the time t after the parachute opened. (10%)

6. Solve the differential equation: (20%)

$$y'' + 4y' + 4y = 7x - 3\cos 2x + 5e^{-2x} + \frac{e^{-2x}}{x}$$

7. Given that $y(x) = 2x$ is a solution of (15%)

$$(8x^2 + 1)y'' - 16xy' + 16y = 0$$

Find the general solution of

$$(8x^2 + 1)y'' - 16xy' + 16y = \left(8x + \frac{1}{x}\right)^2$$

8. Solve the initial value problem: (15%)

$$y'' - 4y' + 4y = f(t); \quad y(0) = 0, \quad y'(0) = 0, \text{ with}$$

$$f(t) = \begin{cases} t & 0 \leq t < 3 \\ t+2 & t \geq 3 \end{cases}$$

