

國立臺灣科技大學102學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科目：工程數學

(總分為100分)

1. Solve for

$$y' - \frac{3}{x} = 5x^2. \quad (20\%)$$

2. Solve for

$$y'' + 9y = 1 + t, \quad y(0) = -1, \quad y'(0) = 1,$$

using only Laplace transform. (20%)

3. Given

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{pmatrix} \quad (20\%)$$

(a) find the inverse matrix A^{-1} , (6%)(b) produce a matrix P that diagonalizes A , also determine $P^{-1}AP$. (14%)4. Let $\vec{F} = -y\hat{i} + xy\hat{j} - xyz\hat{k}$ and let Σ be the part of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \leq 9$. Calculate the surface integral over Σ

$$\iint_{\Sigma} (\nabla \times \vec{F}) \cdot d\vec{A}. \quad (10\%)$$



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5. Solve the following partial differential equation:

$$\begin{aligned}
 u_{xx} + u_{yy} &= 0, & 0 < x < \pi, & & 0 < y < 1 \\
 u(x, 0) &= \sin x \cos x, & 0 \leq x \leq \pi & & \\
 u(x, 1) &= 1, & 0 \leq x \leq \pi & & \\
 u(0, y) &= y, & 0 \leq y \leq 1 & & \\
 u(\pi, y) &= y, & 0 \leq y \leq 1 & &
 \end{aligned}
 \tag{30\%}$$

- (a) What is the type of the second-order partial differential equation given above? (Hyperbolic, parabolic, or elliptic) (2%)
- (b) What is the physical nature of the partial differential equation given above? (wave propagation, equilibrium, or heat/diffusion) (2%)
- (c) What is the type of boundary conditions given above? (Dirichlet, Neumann, or Robin) (2%)
- (d) Is the partial differential equation (PDE) given above a linear PDE? (2%)
- (e) Is the partial differential equation (PDE) given above homogeneous? (2%)
- (f) Obtain the explicit solution of the problem given above. (15%)
- (g) What is the maximum value of $u(x, y)$? (5%)

