

國立臺灣科技大學102學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科 目： 工程數學

(總分為100分)

1. Solve the following ordinary differential equations:

(1) $y' - 3y = e^{3x} \sin(x)$ (10%)

(2) $x^2 y'' - 4xy' + 4y = x^4 + x^2$ (10%)

2. (1) Explain Laplace Transform and describe its advantages (8%)

(2) Use Laplace Transform to solve $y(t)$ from the following differential equation. (12%)

$$ty'' + (4t - 2)y' - 4y = 0$$

With $y(0) = 1; y'(0) = \text{constant}$ 3. (1) Describe the meaning of linear independence for vectors F_1, F_2, \dots, F_k in \mathbb{R}^n . (5%)(2) Evaluate the linear dependence for vectors $12i - 3k, i + 2j - k, -3i + 4j$ in \mathbb{R}^3 . (5%)

國立臺灣科技大學102學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科 目： 工程數學

(總分為100分)

4. Write a series solution for

$$\frac{\partial^2 y}{\partial t^2} = 9 \frac{\partial^2 y}{\partial x^2} - e^{-x} \text{ for } 0 < x < 4, t > 0$$

$$y(0, t) = y(4, t) = 0 \text{ for } t \geq 0$$

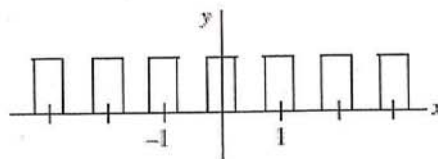
$$y(x, 0) = \sin(\pi x), \quad \frac{\partial y}{\partial t}(x, 0) = 0 \text{ for } 0 \leq x \leq 4. \text{ (15\%)}$$

5. (a) Find the complex Fourier series of the periodic square wave shown in Problem

5. The wave is the periodic extension of the function f :

$$f(x) = \begin{cases} 0, & -\frac{1}{2} < x < -\frac{1}{4} \\ 1, & -\frac{1}{4} < x < \frac{1}{4} \\ 0, & \frac{1}{4} < x < \frac{1}{2} \end{cases} \quad (10\%)$$

- (b) Plot the frequency spectrum of
- f
- in (a). (10%)



Problem 5

6. Let
- $f(z) = \frac{1}{(z-2)(z-1)^3}$
- . Expand
- $f(z)$
- in a Laurent series valid for the

indicated annular domain, $0 < |z-2| < 1$. (15%)