國立臺灣科技大學103學年度碩士班招生試題

系所組別：電機工程系碩士班己二組

科　　目：電磁學

（總分為100分）

Problem 1 (total 15 points)

Non-uniform charge is distributed with a spatial dependence of ρ_0 / r^2 (C/m^3) in the region of $r_0 < r < 1.5r_0$ in spherical coordinate.

(a) Find the electric flux density **D** everywhere. (10%)

(b) Evaluate the potential at $r = 1.5r_0$ if the voltage at infinite is assumed to be zero. The medium is free space. (5%)

Problem 2 (total 30 points)

Shown in Fig. 1, the region x < 0 is filled with Medium 1 while the region x > 0 is filled with Medium 2. Both media extend to infinite. Assume a uniform plane wave having electric field of

$$\mathbf{E}_i = E_0 \sin(10^9 \pi t - 5\pi x)\mathbf{a}_z + E_0 \cos(10^9 \pi t - 5\pi x)\mathbf{a}_y \quad \text{(V/m)}$$

is normally incident from Medium 1 onto the interface $x = 0$. Please answer the following questions:

(a) Determine the linear frequency, phase constant, phase velocity of the incident wave in Medium 1. (5%)

(b) Determine the relative dielectric constant of Medium 1. (5%)

(c) Determine the polarization sense of the incident wave. (5%)

(d) Determine the reflection and transmission coefficients at the interface. (5%)

(e) Express the transmitted wave magnetic field **H** in Medium 2 in phasor form. (5%)

(f) Calculate the <u>time-average power delivered</u> by the <u>incident wave</u> in a square meter area parallel to the yz-plane ($x < 0$) (5%)

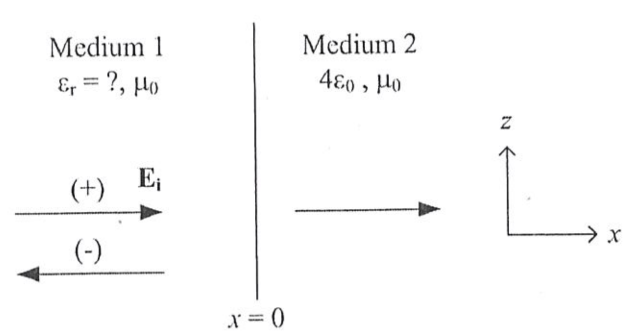

Fig. 1

Problem 3 (total 5 points)

Given a vector field **B**

$$\mathbf{B} = (mxy + nxz^2)\mathbf{a}_x + y^2\mathbf{a}_y - z^3\mathbf{a}_z,$$

find m and n such that **B** can represent a magnetic field in rectangular coordinate system with $|x|, |y|, |z| \le 1$.

國立臺灣科技大學103學年度碩士班招生試題

系所組別： 電機工程系碩士班己二組

科　　目： 電磁學

（總分為100分）

Problem 4 (total 14 points)

For a lossless transmission line system shown in Fig. 2, if a wave carrying 1 Watt of power is incident from the left of Line 1, find

(a) The power reflected back to Line 1; (4%)

(b) The power transmitted to the load resistor of Line 2; (4%)

(c) The power dissipated on the resistor at the junction (2%)

(d) If the load resistance of Line 3 is removed and the Line 3 length is a quarter wavelengths, what is the power reflected back to Line 1. (4%)

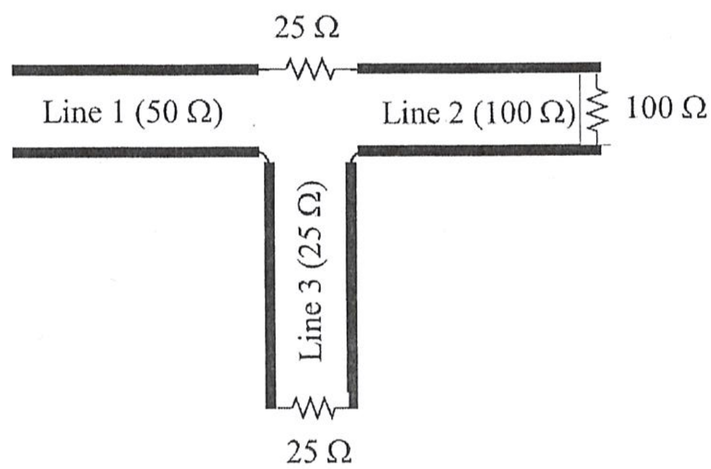

Fig. 2

Problem 5 (total 12 points)

Using following definitions and relationships, find

(a) the gradient of $r^2\cos\theta$ in spherical coordinate. (2%)

(b) the Laplacian of (a) in spherical coordinate. (10%)

$$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}}$$

$$\begin{aligned}\nabla^2\mathbf{A} = &\left(\Delta A_r - \frac{2A_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{r}} \\ &+ \left(\Delta A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{\theta}} \\ &+ \left(\Delta A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{\phi}}\end{aligned}$$

$$\Delta f \equiv \nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$