國立臺灣科技大學 104 學年度碩士班招生試題
系所組別：資訊工程系碩士班
科　　目：計算機數學

總分：100 分

(Show every step of your solution.)

1. (8 points) Determine the number of integer solutions of $x_1 + x_2 + x_3 < 16$, where $x_i \geq 2$ for $3 \geq i \geq 1$.

2. (8 points) Determine the value of positive integer k such that $(7k^3 - 21k^2 + k - 3)$ is a prime number.

3. (8 points) Determine the number of strings in A^3 and A^4, where the alphabet set A is defined as $A = \{v, x, y, z\}$.

4. (8 points) If $(Z_{15}, *)$ is a cyclic group, find all generators of $(Z_{15}, *)$.

5. (8 points) Let $B = \{a, b, c, d, e\}$. Determine the number of relations on B that are reflexive and symmetric.

6. (10 points) Given k matrices A_1, A_2, ..., and A_k, assume the matrix-multiplication-chain $A_1 \times A_2 \times ... \times A_k$ follows the association law.
   (1) (5 points) Write down the recurrence relation for counting the number of ways for calculating the matrix-multiplication-chain $A_1 \times A_2 \times ... \times A_k$.
   (2) (5 points) Solve your derived recurrence relation.

7. (20 points)
   (a) (8 points) Find a basis that spans the plane $x + 2y + z = 0$.
   (b) (7 points) Find the matrix that represents the projection onto the plane $x + 2y + z = 0$.
   (c) (5 points) Find the matrix that represents the reflection of through the plane $ax + by + cz = 0$, where (a, b, c) is a unit vector.

8. (8 points) Mike chooses either pizza or sandwich for lunch. If he chooses pizza for lunch one day, there is a $\frac{4}{5}$ chance that he chooses pizza again the next day. If he chooses sandwich for lunch one day, there is a $\frac{2}{3}$ chance that he chooses pizza the next day. Over the long term, what is the chance that Mike chooses pizza for lunch on any given day?

9. (10 points) Find a curve of the form $y = a + (\frac{b}{x})$ that best fits the data set $\{(2, 3), (1, 4), (4, 1)\}$.

10. (12 points) Let $B_1 = \{(1, 1), (1, -1)\}$ and $B_2 = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$ be bases of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively, and $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \\ 1 & 1 \end{pmatrix}$ be the matrix of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ with respect to B_1 and B_2. Find the matrix of T with respect to the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$.