

## 國立臺灣科技大學 104 學年度碩士班招生試題

系所組別：機械工程系碩士班丁組

科目：系統控制

(總分為 100 分)

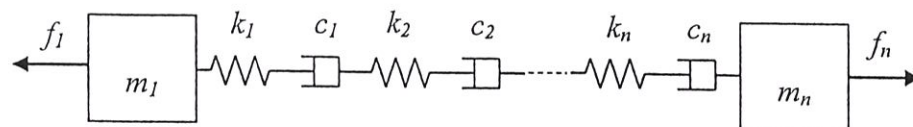
## 1. Modeling of population in Taiwan. (20 point)

It is said that the population policy in Taiwan is facing a severe situation. You are now asked to formulate the problem so that it can be clarified with quantitative evidence to facilitate some proper strategies. A good start for the task is to model the problem in terms of differential equations in which the control activities are able to be enforced. Let us denote the number of individuals as  $x(t) \in \mathfrak{R}$  where we have assumed that the population is large enough so that  $x(t)$  can be viewed as a smooth function of time. To simplify the development, we would like to ignore migration no matter inwards or outwards. The birth rate is denoted as  $b$  which can be used to compute the increase in  $x(t)$  during a short time interval  $\Delta t$  as  $x(t)b\Delta t$ . Likewise, we may denote the death rate as  $d$  which is able to be used to compute the decrease in  $x(t)$  during a short time interval  $\Delta t$  as  $x(t)d\Delta t$ . Suppose the population starts from  $x(t)$  in the above mentioned time interval and ends at  $x(t + \Delta t)$ .

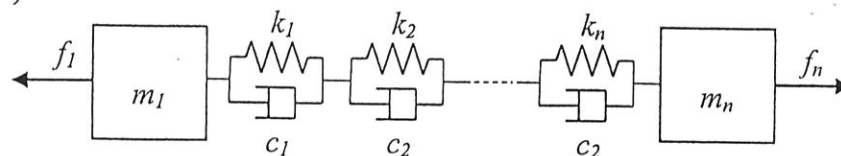
- (a) Give a model of population in Taiwan in difference equation. (10%)  
 (b) Find a differential equation representation by letting  $\Delta t \rightarrow 0$  in (a). (10%)

## 2. Please find the mathematical model of the systems (20 points)

(a) (10%)



(b) (10%)

3. The PID controller widely used in industrial applications is well-known to be in the form of  $u = k_p e + k_i \int e + k_d \dot{e}$ . (10 points)

- (a) Do we need a mathematical model to design the controller? (2%)  
 (b) What is the function for each and every term? (2%)  
 (c) Please state the procedure to tune the gains. (3%)  
 (d) What are the limitations in using this controller? (3%)



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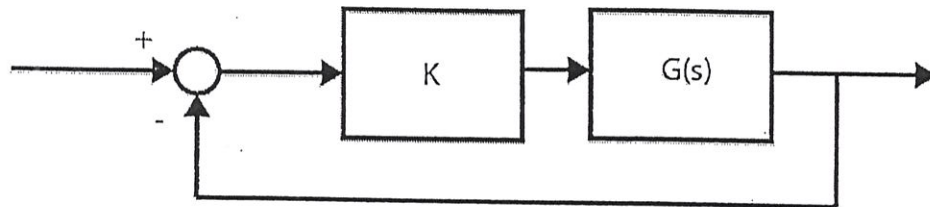
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4. Given two control systems (10 points)

$$\text{System 1: } \begin{cases} \dot{x}_1 = -x_1 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = u \end{cases} \quad \text{System 2: } \begin{cases} \dot{x}_1 = x_1 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = u \end{cases}$$

Are they stabilizable? If yes, design a stabilizing controller. If no, state the reason. (10%)

5. Consider the following block diagram with unity feedback (30 points)



where  $G(s) = \frac{(s-2)}{(s+1)^2}$ , and  $K$  is a constant greater than zero. Please answer the following:

- What is the closed loop transfer function of this system? (5%)
  - Use the Routh's stability criterion to find the range of  $K$  that will stabilize the system. (5%)
  - Please sketch the Nyquist plot of the above system with  $K=1$ , and comment on its stability, compared to the criterion found in (b). (20%)
6. Find the gain margin (5%) and phase margin (5%) associated with the transfer function  $G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$  (10 points)

