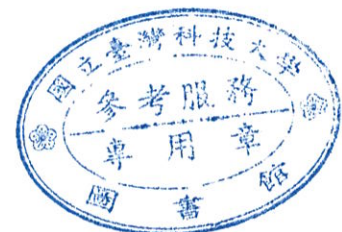


## 國立臺灣科技大學 104 學年度碩士班招生試題

系所組別：自動化及控制研究所碩士班甲組、乙組

科目：工程數學

1. Please evaluate  $\oint_{\gamma} \frac{2iz - \cos(z)}{z^3 + z} dz$ , where  $\gamma$  is a closed path that encloses all singularities of  $\frac{2iz - \cos(z)}{z^3 + z}$ , where  $i = \sqrt{-1}$ . (10%)
2. Please find a differential equation whose general solution is  $y = e^{2x}(x + c_1 \sin(\sqrt{2}x) + c_2 \cos(\sqrt{2}x))$ , where  $c_1$  and  $c_2$  are arbitrary constants. (5%)
3. Please find the Laplace transform of  $t^2 H(t+1)$ , where  $H(t)$  is the Heaviside function (sometimes it is also called the unit step function). (5%)
4. Find the convolution of  $te^{-2t}$  and  $\delta(t-3)$ . (5%)
5. Please find the convergent radius of the Taylor expansion series for  $\frac{x-1}{(x^2+2x+5)}$  about the center point of  $x=2$ . (5%)
6. Let  $\mu(x, y)$  be an integrating factor for  $M(x, y) + N(x, y)y' = 0$ . Suppose that the general solution of this differential equation can be written as  $\Phi(x, y) + c = 0$ . Show that  $\mu(x, y)G(\Phi(x, y))$  is also an integrating factor for the differential equation, where  $G(\ )$  is a differential function of one variable. (10%)
7. Let  $y_1$  and  $y_2$  be two linear independent solution of  $(x^2 - 2x)y'' + 2(1-x)y' + 2y = 0$ . Also we know  $y_1(1) = 1$ ,  $y_1'(1) = 2$ ,  $y_2(1) = 0$  and  $y_2'(1) = 1$ . Please find the Wronskian of  $y_1$  and  $y_2$ . (10%)



## 國立臺灣科技大學 104 學年度碩士班招生試題

系所組別：自動化及控制研究所碩士班甲組、乙組

科目：工程數學

8. Write the Fourier series of the function on the interval and determine the sum of the Fourier series. (15%)

$$f(x) = 1 - |x|, \quad -2 \leq x \leq 2$$

9. Find the reduced form of the matrix  $A$  and use this to determine the rank of the matrix  $A$ . Also find a basis for the row space of the matrix  $A$  and the basis for the column space. (15%)

$$A = \begin{pmatrix} -4 & 1 & 3 \\ 2 & 2 & 0 \end{pmatrix}$$

10. Find the eigenvalues and associated eigenvectors for the given matrix  $B$  below. If the matrix  $B$  is diagonalizable, write a matrix that diagonalizes it. (20%)

$$B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1-i \\ 0 & -1-i & 0 \end{bmatrix}, \text{ where } i = \sqrt{-1}.$$

