國立臺灣科技大學 104 學年度碩士班招生試題

系所組別：自動化及控制研究所碩士班甲組、乙組

科　　目：工程數學

1. Please evaluate $\oint_\gamma \frac{2iz-\cos(z)}{z^3+z}dz$, where $\gamma$ is a closed path that encloses all singularities of $\frac{2iz-\cos(z)}{z^3+z}$., where $i=\sqrt{-1}$. (10%)
2. Please find a differential equation whose general solution is $y=e^{2x}(x+c_1\sin(\sqrt{2}x)+c_2\cos(\sqrt{2}x))$, where $c_1$ and $c_2$ are arbitrary constants. (5%)
3. Please find the Laplace transform of $t^2\,H(t+1)$, where $H(t)$ is the Heaviside function (sometimes it is also called the unit step function). (5%)
4. Find the convolution of $te^{-2t}$ and $\delta(t-3)$. (5%)
5. Please find the convergent radius of the Taylor expansion series for $\frac{x-1}{(x^2+2x+5)}$ about the center point of $x$=2. (5%)
6. Let $\mu(x,y)$ be an integrating factor for $M(x,y)+N(x,y)y'=0$. Suppose that the general solution of this differential equation can be written as $\Phi(x, y)+c=0$. Show that $\mu(x,y)G(\Phi(x,y))$ is also an integrating factor for the differential equation, where $G(\ )$ is a differential function of one variable. (10%)
7. Let $y_1$ and $y_2$ be two linear independent solution of $(x^2-2x)y''+2(1-x)y'+2y=0$. Also we know $y_1(1)=1$, $y_1'(1)=2$, $y_2(1)=0$ and $y_2'(1)=1$。Please find the Wronskian of $y_1$ and $y_2$. (10%)

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8. Write the Fourier series of the function on the interval and determine the sum of the Fourier series. (15%)

$f(x)=1-|x|,\ -2\le x\le 2$

9. Find the reduced form of the matrix $A$ and use this to determine the rank of the matrix $A$. Also find a basis for the row space of the matrix $A$ and the basis for the column space. (15%)

$$A=\begin{pmatrix}-4 & 1 & 3\\ 2 & 2 & 0\end{pmatrix}$$

10. Find the eigenvalues and associated eigenvectors for the given matrix $B$ below. If the matrix $B$ is diagonalizable, write a matrix that diagonalizes it. (20%)

$$B=\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 1-i\\ 0 & -1-i & 0\end{bmatrix},\ \text{where}\ \ i=\sqrt{-1}.$$