

## 國立臺灣科技大學 105 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組

科目：作業研究

(總分為 100 分)

1. (30%) Consider the following problem.

$$\text{Maximize } Z = 7x_1 + 4x_2 + 2x_3$$

subject to

$$3x_1 + 2x_2 + 3x_3 \leq 30$$

$$2x_1 + x_2 + x_3 \leq 30$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

- (1) Construct the dual problem for this primal problem. (10%)  
 (2) Solve the dual problem. (10%)  
 (3) Use the optimal solution to the dual problem obtained in part (2) and the complementary slackness theorem to derive the optimal solution to the primal problem. (10%)
2. (20%) Use the branch-and-bound algorithm to solve the following problem.

$$\text{Minimize } Z = 3x_1 + 2x_2$$

subject to

$$x_1 + x_2 \geq 3$$

$$3x_1 + x_2 \geq 6$$

$$x_1 \geq 0, x_2 \geq 0$$

$$x_1, x_2 \text{ are integers}$$

3. (15%) There are two firms producing widgets. It costs the first firm  $q_1$  dollars to produce  $q_1$  widgets and the second firm  $0.5q_2^2$  dollars to produce  $q_2$  widgets. If a total of  $q$  widgets are produced, consumers will pay  $\$200 - q$  for each widget. If the two manufacturers want to collude in an attempt to maximize the sum of their profits, how many widgets should each company produce?
4. (15%) Machines in a factory break down at an exponential rate of six per hour. There is a single repairman who fixes machines at an exponential rate of eight per hour. The cost incurred in lost production when machines are out of service is  $\$10$  per hour per machine. What is the average cost rate incurred due to failed machines?
5. (20%) The NTUST television network earns an average of  $\$400,000$  from a hit show and loses an average of  $\$100,000$  on a flop. Of all shows reviewed by the network, 25% turn out to be hits and 75% turn out to be flops. For  $\$40,000$ , a market research firm will have an audience view of a prospective show and give its view about whether the show will be a hit or a flop. If a show is actually going to be a hit, there is a 90% chance that the market research firm will predict the show to be a hit. If the show is actually going to be a flop, there is an 80% chance that the market research firm will predict the show to be a flop. Determine how the network can maximize its expected profits.

