國立臺灣科技大學 105 學年度碩士班招生試題

系所組別：材料科學與工程系碩士班乙組

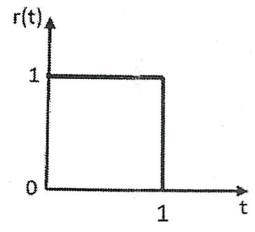
科　　目：工程數學

(總分為 100 分)

1. **(30 points)** Solve the initial value problem

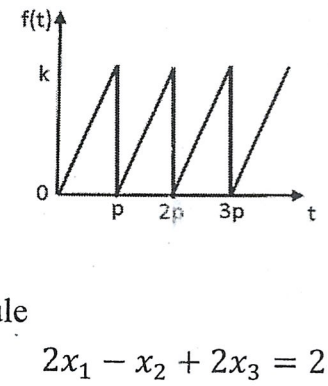
   (a) $y'' + y' - 2y = 0$, $y(0) = 4$, $y'(0) = 1$ **(10 points)**

   (b) $y'' + 2y = r(t)$, $y(0) = 0$, $y'(0) = 0$, where r(t) = 1 if $0 < t < 1$ and 0 otherwise. **(20 points)**



2. **(10 points)** Find the Laplace transform of the function

$$f(t) = \frac{k}{p}t \qquad when\ 0 < t < p, \quad f(t+p) = f(t)$$



3. **(10 points)** Solve by Cramer's rule

$$2x_1 - x_2 + 2x_3 = 2$$
$$x_1 + 10x_2 - 3x_3 = 5$$
$$-x_1 + x_2 + x_3 = -3$$

4. **(10 points)** Find an equation of the plane that contains (1, 2, -1), (4, 3, 1), and (7, 4, 3).

5. **(10 points)** Compute A^m for $A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$, m= 3

6. **(10 points)** Find the Fourier series of f on the given interval.

$$f(x) = \begin{cases} 1, & -5 < x < 0 \\ 1 + x. & 0 \le x < 5 \end{cases}$$

7. **(10 points)** The temperature at a point (x, y) on a rectangular metal plate is given by $T(x,y) = 100 - 2x^2 - y^2$. Find the path a heat-seeking particle will take, starting at (3, 4), as it moves in the direction in which the temperature increases most rapidly.

8. **(10 points)** Compute z^3 for $z = 1 - \sqrt{3}i$