國立臺灣科技大學105學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科　　目：工程數學

(總分為 100 分)

1. Consider the following second order constant-coefficient ordinary differential equation:

$$a\ddot{y} + b\dot{y} + cy = 3, \qquad y(0) = 1 \text{ and } \dot{y}(0) = 2$$

(a). If $a = 1, b = 3, c = 2$, find the solution $y(t) = ?$ (15%)

(b). If $a = 1, c = 2$, and b is a tunable parameter, what will be the condition for b in order to get an ***oscillatory*** solution with ***decaying amplitude***? (5%)

2. Use the method of ***Laplace transform*** to solve the following initial value problem :

$$\ddot{y} + y = \frac{1}{a}[U(t) - U(t-a)], \qquad y(0) = 0 \text{ and } \dot{y}(0) = 0,$$

where $U(t)$ is the ***unit step function*** defined as follows:

$$U(t) = \begin{cases} 0 & t < 0, \\ 1 & t \geq 0. \end{cases}$$

What will be the solution if $a \to 0$? (20%)

3. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$, answer the following questions:

(a). ***Diagonalize*** the matrix A. (10%)

(b). Solve the following system of ordinary differential equations

$$\frac{d}{dt}\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = A\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$$

subject to the following initial conditions: (10%)

$$\begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$$

4. By evaluating both sides of the equation, verify Stokes' theorem for the vector field $\vec{F} = 4z\vec{i} - 2x\vec{j} + 2x\vec{k}$ over the plane of intersection of the cylinder $x^2 + y^2 \leq 1$ and the plane $z = y + 1$. (20%)

5. Starting from separation of variables, solve the boundary-value problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 \leq r \leq 2, \ 0 \leq z \leq 1$$

$$u(r,0) = 0, \ u(r,1) = 1, \ u(2,z) = 0.$$ (20%)