

國立臺灣科技大學 107 學年度碩士班招生試題

系所組別：資訊工程系碩士班

科目：計算機數學

(總分為 100 分)

1. (10%) In a group of 20 people who join a lunch party, we know the sum of their ages is 380 and the youngest one has his/her age of 10. Show that there are at least two people who are at the same age.

2. (15%) In a class of 100 students that consists of 40% girls. We also know that there are 16 students who were born in February and half of them are female. A discrete math exam has been held recently and 65 students passed the exam in the end. In the group that passed the exam, $1/13$ were born in February and 35 of them are girls. After all, we have 3 female students who passed the exam and were born in February.

(a) (8%) How many boy students in the class who passed the exam and were not born in February?

(b) (7%) On the side, we also know some other information about the class as follows:

(i) A total of 6 boys received A in the exam and only one of them was born in February.

(ii) There are record-breaking 3 February-born students received A in the exam.

(iii) The percentage of getting A for those students who passed the exam is 20%. Moreover, the percentage makes no difference between the male and female students.

Now a generous buffet restaurant offers a free meal for girl students (because they may eat not much) who were born in February and received A in the discrete math exam. What is the probability for a student in the class to get this free meal?

3. (15%) A (rooted) tree can be recursively defined as follows:

BASIC STEP: A single vertex r is a tree.

RECURSIVE STEP: Suppose that T_1, T_2, \dots, T_n are trees with roots r_1, r_2, \dots, r_n , respectively. Then the graph formed by starting with a root r and an edge from r to each of the vertices r_1, r_2, \dots, r_n , is also a tree.

(a) (8%) Use the similar format to write down the recursive definition of *binary search tree*.

(b) (7%) The same question again, but for the *complete* binary tree that contains the maximum number $2^k - 1$ of nodes with depth $k \geq 0$, such as the following tree of depth 4 in Figure 1. Note that a tree of a single node is considered a tree of depth 1.

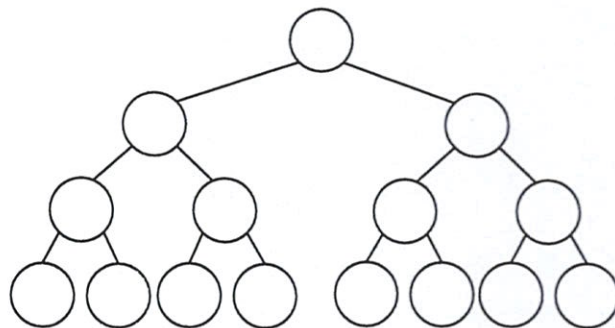
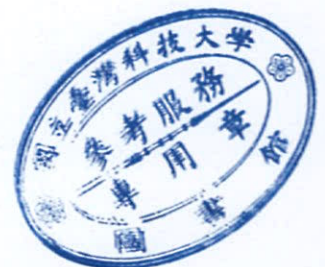


Figure 1



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4. (10%) Let $P(n)$ denote the number of ways to pay the amount n by using the bills of 1, 5, 10, 20, 50, or 100. Show how to obtain $P(300)$ if we have a software that can compute function derivatives at any order. For instance, we can use the software to compute

$$f^{(k)}(x) = \frac{d^k f(x)}{dx^k}$$

for any given $k > 0$.

5. (30%, 3% each) True or False. (No proof needed.)

- Let V be a vector space and W be one of V 's proper subspace. Then $\dim W < \dim V$.
- Let V be a vector space. Then there exist $S_1 \subseteq V$ and $S_2 \subseteq V$ such that $\text{span}(S_1 \cap S_2) \subset \text{span}(S_1) \cap \text{span}(S_2)$.
- Let T be a linear operator on a finite dimensional vector space V . Then for any two bases B_1 and B_2 of V , $[T]_{B_1}$ is similar to $[T]_{B_2}$.
- Let A be a square matrix. Then any two different eigenvectors of A are linearly independent.
- There exists a diagonalizable matrix that is not invertible.
- Let V be a vector space and S be an arbitrary finite subset of V . By applying the Gram-Schmidt orthogonalization process, we can construct an orthonormal set S' from S such that $|S'| = |S|$.
- Every symmetric matrix with real entries is diagonalizable.
- Let X and Y be the eigenspaces corresponding to distinct eigenvalues of a matrix A . Then for any two vectors $x \in X$ and $y \in Y$, x is orthogonal to y .
- There exists a linear system of three variables that has exactly three solutions.
- Let A be an $m \times n$ matrix, where $\text{rank}(A) < n$. Then $x = (A^T A)^{-1} A^T b$ is the least squares solution of the linear system $Ax = b$.

6. (10%) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}$. Find the QR -decomposition of A .

7. (10%) Let A be an $n \times n$ matrix. Prove that $\text{rank}(A) = 1$ if and only if there exist two $n \times 1$ vectors x and y such that $A = xy^T$.

