國立臺灣科技大學 107 學年度碩士班招生試題

系所組別：材料科學與工程系碩士班乙組

科　　目：工程數學

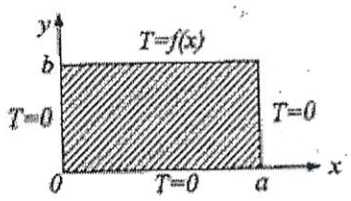
(總分為 100 分)

1. Consider the family of curves that are graphs of
   $F(x, y, K) = y - Kx^2 = 0$
   for various value of K. This is a family of parabolas. Please find the family of orthogonal trajectories. (10%)

2. $f_n(x)$ is a polynominal of order n (n = 0, 1, 2 ….) and these polynomials are mutually orthogonal on the range 0 to ∞, with the weight function e^{-x} that is,
   $\int_0^\infty e^{-x} f_n(x) f_m(x) = 0$ if m≠n
   Find a differential equation satisfied by $f_n(x)$ of the from $x\frac{d^2 f_n}{dx^2} + g(x)\frac{df_n}{dx} + \lambda_n f_n = 0$
   (15%)

3. solve the following different equation (10%):
   $xy''(x)+y'(x)+\frac{1}{4}y(x) = 0$, hint: let $\sqrt{x}=z$

4. When the temperature of a solid is independent of time, it satisfies the equation:
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$
   For a slab, $0 \le x \le a$, $0 \le y \le b$, $|z| \le h$, at steady state, the temperature at boundaries is shown in the figure. Assuming that the faces $z = \pm h$ are insulated, please find T (x, y)
   (15%)



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5. Find the Fourier series of f on the given interval. (10%)

$$f(x)=\begin{cases}0, & -\pi/2<x<0\\ \cos x, & 0\le x<\pi/2\end{cases}$$

6. Solve $X'=\begin{pmatrix}-1 & 3 & 0\\ 3 & -1 & 0\\ -2 & -2 & 6\end{pmatrix}X$ by diagonalization. (10%)

7. Find the volume of the solid bounded by the graphs of the giving equations. $z = 1 + x^2 + y^2$, $3x + y = 3$, $x = 0$, $y = 0$, $z = 0$ (first octant) (20%)

8. Finding the eigenvalues and eigenvectors of the given matrix. (10%)

$$\begin{pmatrix}1 & 2 & 3\\ 0 & 5 & 6\\ 0 & 0 & -7\end{pmatrix}$$