

國立臺灣科技大學 107 學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科目：工程數學

(總分為 100 分)

1. (20%) Given an ODE: $y''' + 3y'' + 3y' + y = 30e^{-x}$,
- (a) Find the **homogeneous** solution. (10%)
- (b) Find the **particular** solution. (10%)

2. (20%) Solve the following ODE by using the Laplace transform,

$$y'' + 2y' + y = 3te^{-t}, \quad y(0) = 4, \quad y'(0) = 2.$$

3. (20%) Given an ODE: $y'' + 15y' + 50y = 0$. Convert it into a system of the form: $y' = Ay$,

- (a) Find the matrix **A**. (4%)
- (b) Find the **eigenvalues** and the corresponding **eigenvectors** of **A**. (8%)
- (c) Find the **general** solution to the system $y' = Ay$. (8%)

4. (20%) Let $\vec{F}(x, y, z) = (-2y, -x + y, 0)$ be a force field in 3D space and $C: x^{1/2} + y^{1/2} = 1, z = 0, 0 \leq x \leq 1$ be a curve on the $x - y$ plane.

- (a) Determine if the force field $\vec{F}(x, y, z)$ is **conservative** or **not conservative**. (4%)
- (b) Calculate the work done by the force field \vec{F} along the curve C directly, *i.e.*, $\int_C \vec{F} \cdot d\vec{R}$. (8%)
- (c) Verify the result you just obtained by applying the **Stokes' theorem**. Note that C is NOT a closed curve. (8%)

5. (20%) Consider the following 2nd order partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, 0 < y < \pi,$$

subjected to the following boundary conditions:

$$\begin{aligned} \frac{\partial u}{\partial x}(0, y) &= 0, & 0 \leq y \leq \pi \\ \frac{\partial u}{\partial x}(\pi, y) &= 0, & 0 \leq y \leq \pi \\ \frac{\partial u}{\partial y}(x, \pi) &= 0, & 0 \leq x \leq \pi \\ \frac{\partial u}{\partial y}(x, 0) &= \cos x, & 0 \leq x \leq \pi \end{aligned}$$

- (a) Determine if this is a **Dirichlet** type, **Neumann** type, or **Robin** type boundary value problem? (2%)
- (b) Solve this boundary value problem by the **method of separation of variables** in details. (12%)
- (c) How will the solution become if the $\cos x$ in the 4th boundary condition is changed to $\sin x$? (6%)

