國立臺灣科技大學 109 學年度碩士班招生試題
系所組別：機械工程系碩士班丁組
科　　目：系統控制

(總分為 100 分)

題目共六大題，總分 100 分，每小題有標示所占分數

1. Answer the followings:

(1) (multiple answers) If the dominant complex pole of a closed-loop system is moved along a horizontal line on the s-plane, which of the followings will remain almost un-changed? (a) peak time of step response (b) damping ratio (c) settling time of step response (d) damped natural frequency (e) maximum overshoot of step response. (5%)

(2) (multiple answers) Which of the followings are valid statements: (a) the transfer function is defined as the output Laplace Transform divided by the input Laplace transform (b) the break frequency on the Bode magnitude plot of a first order system is the same as the inverse of the time constant (c) marginally stable is equivalent to bounded-input, bounded-output (BIBO) stable (d) the max. overshoot in the step response of a standard second order system depends on both the damping ratio and the natural frequency (e) on the root locus, the determination of the real-axis segment is based on the phase condition. (5%)

(3) Consider the unity feedback structure defined below, if $G_e(s)$ is a standard second order system ($G_e = \frac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$) with K, ζ and ω_n positive, use root locus, Bode plot, and Routh table to explain why the closed-loop system is always stable. (all three of them) (6%)

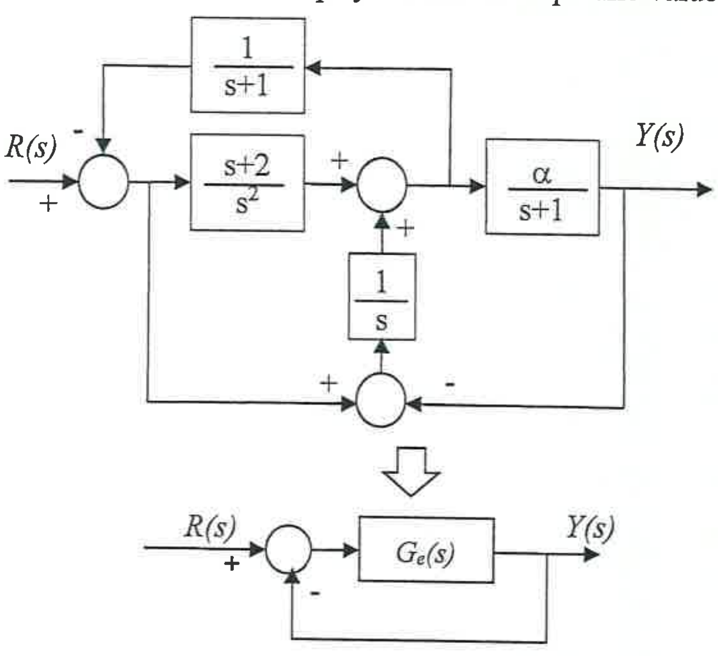
2. For the system shown below, simplify the block diagram to standard unity feedback structure.

(1) Derive the equivalent transfer function $G_e(s)$. (7%)

(2) Determine the system type and the corresponding error constant. (4%)

(3) Find the range of α such that the closed-loop system is stable. (4%)

(4) If the closed-loop system is marginally stable for a certain value of α, what are the imaginary axis roots of the closed-loop system for that specific value of α. (4%)



Standard unity feedback system structure

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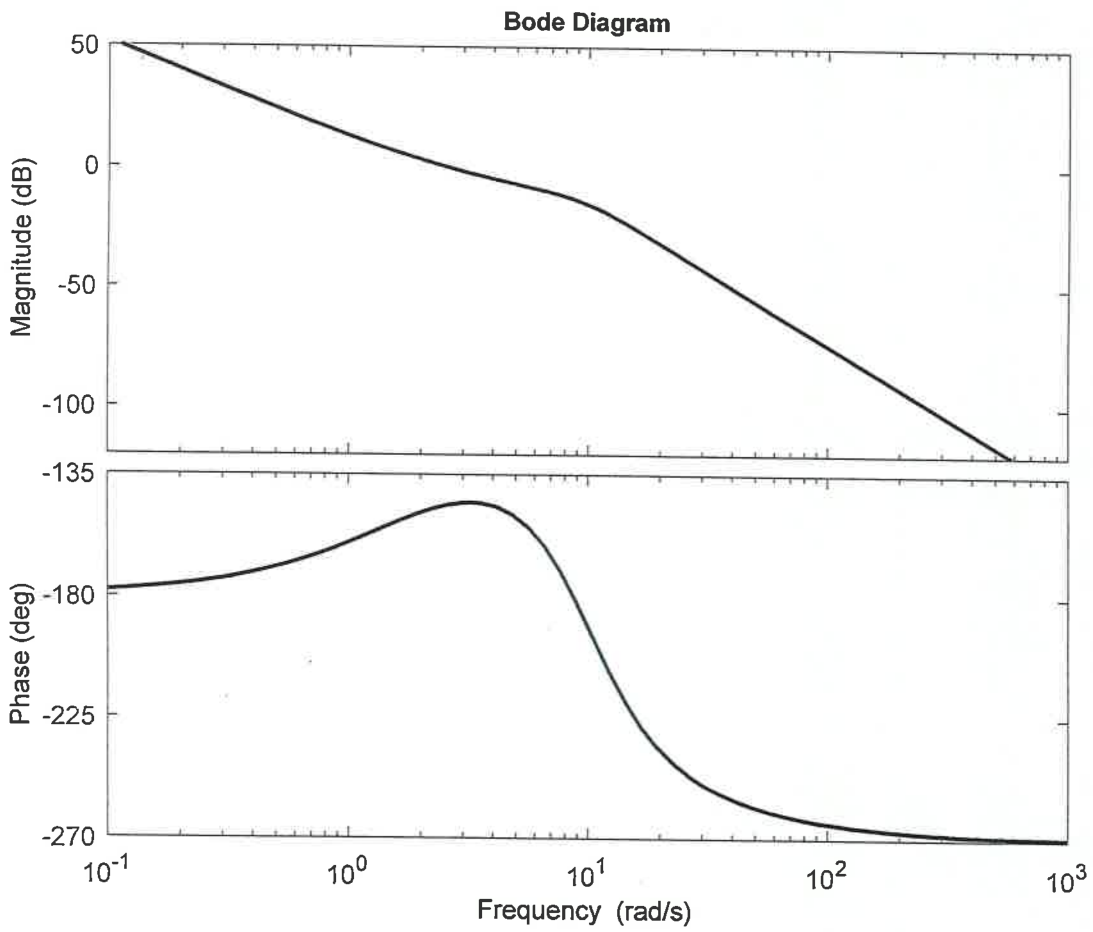
3. Below is the Bode plot of an open loop transfer function $G_e(s)$ under unity feedback structure, assume that $G_e(s)$ is minimum phase with no pole or zero in the open right half plane. Answer the followings:

   (1) What is the system type? (3%)

   (2) How many asymptotes should there be on the root locus? (3%)

   (3) Determine the phase margin and gain margin of the system. (4%)

   (4) Roughly estimate the location of the zero of the closed-loop system. (3%)



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4. Consider a single input linear time invariant system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$ where $\mathbf{x} = [x_1 \quad \cdots \quad x_n]^T \in \Re^n$ is the state vector, and $u \in \Re$ is the control input. The matrices $\mathbf{A} \in \Re^{n\times n}$ and $\mathbf{b} \in \Re^n$ are known.

(a) Design a state feedback controller so that $x_1 \to 0$ as $t \to \infty$. (4%)

(b) Design a state feedback controller so that $x_1 \to 1$ as $t \to \infty$. (4%)

(c) Compute the steady state x_1 for the system with the controller you designed in (a). (4%)

(d) Compute the steady state x_1 for the system with the controller you designed in (b). (4%)

5. Given a stable LTI plant with the transfer function

$$G_P(s) = \frac{k}{s^2 + a_1 s + a_0}$$

where k, a_1, a_0 are known constants. A PID controller is to be designed so that the unity feedback closed loop system behaves like a target 2nd order LTI system with damping ratio ζ and natural frequency ω_n. The transfer function of the PID controller is constructed in the form

$$G_{PID}(s) = k_p\left(1 + \frac{1}{T_i s} + \frac{Ns}{s + \frac{N}{T_d}}\right) = \frac{k_p\left[s^2 + \left(\frac{N}{T_d} + \frac{1}{T_i} + 1\right)s + \frac{N}{T_i T_d}\right]}{s\left(s + \frac{N}{T_d}\right)}$$

We would like to determine the parameters k_p, T_i, T_d and N by following steps:

(a) Compute the open-loop transfer function $G_{OL}(s) = G_{PID}(s)G_P(s)$. (4%)

(b) Compare the coefficient of the denominator of $G_P(s)$ with the numerator of $G_{PID}(s)$, and you can cancel some terms out so that two equations are obtained. (4%)

(c) Compute the closed-loop transfer function $G_{CL}(s) = \dfrac{G_{OL}(s)}{1 + G_{OL}(s)}$. (4%)

(d) Compare the closed-loop transfer function with the target 2nd order system to get two more equations. (4%)

With these four equations, we have enough information to determine all parameters in the PID controller, and we are done for the design.

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6. A spring scale used for weighing is modeled as in the schematic below. A pan (with mass m) is used to support the object to be weighed (with mass M). A rack is attached to the pan so that the displacement ($x(t)$) can be visualized through a set of gears and the pointer. The pointer is modeled as a rotational inertia (J_2) connected to the gears via a spring (K_2). For simplicity we shall assume that the mass of the pan is negligible (m=0), and we will use the radii of the three gears, i.e., r_1, r_2 and r_3, to represent their corresponding size and the number of teeth. The two small gears are assumed to be massless.

(1) Write down **all the governing equations** to fully describe the system, including the translational and rotational dynamics. (6%)

(2) Determine the **degree of freedom**, system **order** (i.e., the number of poles of the transfer function), and **how many state variables** are necessary to fully describe the system. (3%)

(3) **Find the transfer function** of the system $\frac{\theta_2(s)}{M(s)}$, since there are many symbols involved, you do NOT need to reduce the fraction to its simplest form. (5%)

(4) Suppose that the object to be weighed is a human, if the subjects being weighed complain about the excessive oscillation causing nausea, **explain** how you could **modify the parameters** to help alleviate the situation. (answers without reasoning will not be graded) (3%)

(5) An experienced control engineer criticizes the modelling, saying that the pointer will never rest and a readout is not possible. Give your reasoning about **whether to agree** with his statement or not. (3%)