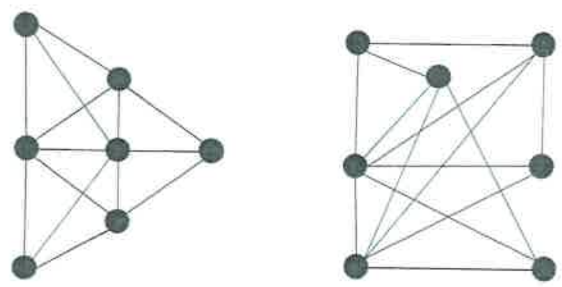
國立臺灣科技大學110學年度碩士班招生試題
系所組別：資訊工程系碩士班
科　　目：計算機數學

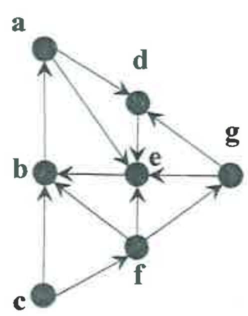
(總分為 100 分)

1. (24%, 3% each) **True** or **False**. (No proof needed.)
   (a) The set of rational numbers between 0 and 1 (inclusive) is a countably infinite set.
   (b) The Boolean product of $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ is $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$.
   (c) $C_k^{k+1} + C_k^{k+2} + \cdots + C_k^{k+n} = C_{k+1}^{k+n+1}$.
   (d) $(\neg q \vee r) \wedge \neg p \leftrightarrow \neg(((\neg p \rightarrow \neg r) \wedge q) \vee p)$ is a tautology, where p, q, and r are propositional variables.
   (e) The following two graphs are nonisomorphic.



   (f) There exists an Euler circuit in each complete bipartite graph.
   (g) There exists a Hamilton circuit in each complete bipartite graph.
   (h) $\forall x P(x) \rightarrow \forall x Q(x) \equiv \forall x (P(x) \rightarrow Q(x))$, where P(x) and Q(x) are predicates.

2. (6%, 2% each) Answer the following questions according to the graph below.



   (a) Is this graph weakly connected?
   (b) Is this graph strongly connected?
   (c) How many strongly connected components are there in this graph? Please also provide all strongly connected components of this graph.

國立臺灣科技大學 110 學年度碩士班招生試題
系所組別：資訊工程系碩士班
科　　目：計算機數學

3. (20%, 5% each) In the standard poker, there are 52 cards with four suits (diamond, club, heart and spade) where each suit contains 13 cards, which are numbered as 1 ~ 10, 11 (J), 12 (Q), and 13 (K).
   (a) How many different ways can we distribute these cards to three players where each player will get five cards?
   (b) Please calculate the minimum number of cards that we need to take from the standard deck of 52 cards to ensure the probability of obtaining at least two cards with the same suit exceeds 0.7.
   (c) Suppose that we pick a card randomly from the standard deck of 52 cards five times and each time the picked card is returned to the deck before the next pick. What is the probability of getting exactly two times of "diamond"?
   (d) Suppose that we pick a card randomly from the standard deck of 52 cards five times and each time the picked card is returned to the deck before the next pick. A random variable X is defined as the number of times that a red card (diamond or heart) is obtained. Please calculate the distribution of this random variable.

4. (10%, multiple choice question, no proof needed, points will only be awarded if all correct answers are selected) The singular value decomposition (SVD) can be leveraged to factorize a real value matrix A into a product of three simpler matrices $A = U\Sigma V^T$, where T denotes transpose operator. Which of the following statements are correct about the SVD decomposition?
   (a) $UU^T = I$
   (b) $U^TU \neq I$
   (c) $VV^T \neq I$
   (d) $V^TV = I$
   (e) $V^TA^T = \Sigma U^T$
   (f) $\Sigma V^T = AU^T$
   (g) The result of performing SVD for a given matrix is unique
   (h) $\text{rank}(A) = \text{rank}(\Sigma)$
   (i) The column vectors of V are eigenvectors of A^TA
   (j) The row vectors of U are eigenvectors of AA^T

國立臺灣科技大學 110 學年度碩士班招生試題

系所組別：資訊工程系碩士班

科　　目：計算機數學

5. (10%) Please find a set of bases for the null space of matrix $\begin{bmatrix} 5 & 2 & 3 & 4 & 5 \\ 1 & 4 & 0 & 0 & 4 \\ 1 & 0 & 3 & 0 & 3 \\ 2 & 0 & 0 & 2 & 2 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 5 & 4 & 3 & 2 \end{bmatrix}$.

6. (10%, multiple choice question, no proof needed, points will only be awarded if all correct answers are selected) Which numbers are eigenvalues of matrix $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$?
   (a) 0
   (b) 1
   (c) −1
   (d) 2
   (e) −2
   (f) 5
   (g) −5

7. (10%, multiple choice question, no proof needed, points will only be awarded if all correct answers are selected) About the determinant of matrices $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times n}$, which of the statements are correct?
   (a) If $AA^{-1} = I$, then $\det(A)\det(A^{-1}) = 1$
   (b) $\det(AB) = \det(BA)$
   (c) $\det(BA) = \det(A)\det(B)$
   (d) $\det(A^7) = (\det(A))^7$
   (e) $\det(A + B) = \det(A) + \det(B)$
   (f) If α is a constant, then $\det(\alpha A) = \det(\alpha)\det(B) = \alpha\det(A)$

8. (10%) The Cholesky decomposition can factorize a symmetric positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose. For a given matrix $A = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 3 & 2 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 9 & 0 \\ 1 & 0 & 0 & 0 & 4 \end{bmatrix}$, please find a lower triangular matrix L by using the Cholesky decomposition so that $A = LL^T$, where T denotes transpose operator.