

國立臺灣科技大學 110 學年度碩士班招生試題

系所組別：自動化及控制研究所碩士班

科目：工程數學

(總分為 100 分)

1. Solve the following ordinary differential equations:

$$(1) y' + e^{-x}y^2 - y = e^x \quad (10\%)$$

$$(2) y'' - 4y' + 13y = 3e^{2x} - 5e^{3x} \quad (10\%)$$

2. Find the recurrence relation and use it to generate the first five terms of a power series solution about 0.

$$y'' - xy' + y = 3 \quad (10\%)$$

3. (1) Two vectors are given by $A = 3i - 6j - 2k$ and $B = 2i + 3.8j + 9k$ from the original. Calculate the angle between A & B and get the unit vector which is perpendicular to both vectors. (6%)

(2) Evaluate the linear dependence for vectors $3i + 6j + k$, $4i + 2j - 4k$, $i - j + k$ in \mathbb{R}^3 and state the reason. (4%)

4. Determine the Fourier cosine transform and Fourier sine transform of the following function.

$$f(t) = \begin{cases} \cos(t) & \text{for } 0 \leq t \leq K \\ 0 & \text{for } t > K \end{cases}, \text{ if } K \text{ is any positive number.} \quad (10\%)$$



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5. Solve the telegraph equation

$$\frac{\partial^2 u}{\partial t^2} + A \frac{\partial u}{\partial t} + Bu = c^2 \frac{\partial^2 u}{\partial x^2}$$

for $0 < x < L, t > 0$. A and B are positive constants.

The boundary conditions are

$$u(0,t) = u(L,t) = 0 \quad \text{for } t \geq 0,$$

and the initial conditions are

$$u(x,0) = f(x), \quad \frac{\partial u(x,0)}{\partial t} = 0 \quad \text{for } 0 \leq x \leq L,$$

Assume that $A^2 L^2 < 4(BL^2 + c^2 \pi^2)$.

(20%)

6. Use convolution to find the inverse Fourier transform of the function.

$$\frac{\sin 3\omega}{\omega(2+i\omega)}$$

(10%)

7. Evaluate

$$\oint_{\gamma} \frac{z}{2+z^2} dz$$

with γ as the circle $|z| = 2$ first by using the residue theorem and then by using the argument principle.

(10%)

8. How many different codes can be formed from the lower-case letters (a - z) of the English alphabet, if a code consists of **seventeen** distinct letters, with different orders counted as different codes.

(10%)

