

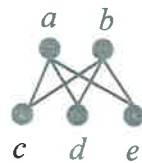
## 國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：資訊工程系碩士班

科目：計算機數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. Which of the following statements are correct? (14%)
  - A. An undirected graph has an odd number of vertices of odd degree. (2%)
  - B. Let  $f$  be the function from  $\mathbb{N}$  to  $\mathbb{N}$  such that  $f(x) = x + 1$ , where  $\mathbb{N}$  is the set of natural numbers. (2%)  
 $f$  is a one-to-one correspondence.
  - C.  $Q_n$  is bipartite. (Hint: An  $n$ -dimensional hypercube, or  $n$ -cube, denoted by  $Q_n$ , is a graph with  $2^n$  vertices representing all bit strings of length  $n$ , where there is an edge between two vertices that differ in exactly one bit position.) (2%)
  - D. The following compound proposition is satisfiable. (2%)  
 $((p \vee q) \rightarrow r) \wedge (\neg p \rightarrow \neg r) \wedge ((p \leftrightarrow r) \rightarrow \neg(q \leftrightarrow r))$
  - E. Suppose that the domain consists of all creatures, and let  $P(x)$  and  $Q(x)$  be the propositional functions “ $x$  is a cat” and “ $x$  is cute,” respectively. The sentence “every cat is cute” can be translated as  $\forall x (P(x) \wedge Q(x))$ . (2%)
  - F. For all sets  $A$ ,  $B$ , and  $C$ ,  $\overline{(A \cup B)} \times \bar{C} = \overline{(A \cap B)} \times \bar{C}$ . (2%)
  - G.  $\mathcal{P}(\{\emptyset\}) = \{\emptyset\}$ . (Hint: The power set of a set  $S$ , denoted as  $\mathcal{P}(S)$ , is the set of all subsets of  $S$ .) (2%)
2. Please answer the following questions about graphs. (16%)
  - (1) What is the sum of the degrees of all vertices in a minimum spanning tree with  $k$  nodes? (6%)
  - (2) How many distinct paths of length  $k$  are there from vertex  $a$  to vertex  $a$  in the following graph? (10%)



3. How many ways are there to distribute 15 indistinguishable cookies to three children? Note that everyone should receive at least one cookie. (10%)
4. A club consists of members from three cities. 20% of its members are from City  $A$ , 40% of its members are from City  $B$ , and 40% of its members are from City  $C$ . Suppose that 15% of members from City  $A$  are taller than 180 cm, 2.5% of members from City  $B$  are taller than 180 cm, and 10% of members from City  $C$  are taller than 180 cm. If a member is taller than 180 cm, what is the probability that this person is from City  $C$ ? (10%)



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5. Let  $A = \begin{bmatrix} 1 & 2 & -6 \\ -3 & 4 & 7 \\ 2 & 4 & 3 \end{bmatrix}$  (24%)

- (1) Please find the determinant of A. (5%)
- (2) Is A singular or nonsingular? (4%)
- (3) Please find the LU decomposition of A. (10%)
- (4) Let  $M$  be a  $3 \times 3$  matrix and the determinant of  $M$  is 4. Please evaluate the determinant for  $(5M^{-1})^T$ . (5%)

6. Let  $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$  be an ordered basis for  $\mathbb{R}^3$ . Please find the coordinates of  $[7 \ -3 \ 4]^T$  with respect to B. (6%)

7. Let  $\mathcal{V}$  be a subspace of  $\mathbb{R}^3$  spanned by the orthogonal basis  $B = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ . Please find the closest point in  $\mathcal{V}$  to the point  $[1 \ 2 \ 3]^T$ . (10%)

8. Let  $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$ . Please find all the eigenvalues of A. (10%)

