國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：電子工程系碩士班丙組

科　　目：電磁學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

For your reference:

(1) The Laplace's equation in spherical coordinates is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial V}{\partial\phi}\right)+\frac{1}{r^2\sin^2\phi}\frac{\partial^2 V}{\partial\theta^2}=0$$

(2) For $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$, n=0, 1, 2,....., we have $y_n=P_n(x)$ as

$$P_0(x)=1,\quad P_1(x)=x,\quad P_2(x)=\frac{1}{2}(3x^2-1),\quad P_3(x)=\frac{1}{2}(5x^3-3x)$$

1. (18%) Please write down: (1) the differential form of the four Maxwell's equations (12%);
   (2) the Lorentz's force equation (3%);
   (3) the equation of continuity (3%).

2. (22%) The initial electric field intensity of the space (vacuum) is $\bar{E}_{initial}=E_0\bar{a}_z$. Now an uncharged conducting sphere (with radius of R) is added as shown in Fig. 1. Please:
   (1) write the relations between the Cartesian and spherical coordinates (where r is the distance from origin, ϕ is the angle relative to the (+z) axis, θ is the angle relative to the (+x) axis towards the (+y) axis on the xy plane) (5%);
   (2) derive the final potential (V) distribution of the space (assume V(0,0,0)=0) (9%);
   (3) derive the final electric field intensity ($\bar{E}$) of the space (4%);
   (4) derive the final charge distribution (ρ) on the sphere (4%).

   (Please use spherical coordinates for (2)~(4))

3. (10%) As shown in Fig. 2, an infinitely long wire with radius R has uniform electron distribution (with number per unit volume be ρ) and the electrons are flowing in speed of $\bar{v}=|\bar{v}|\hat{a}_z$.

   Determine the magnetic flux density of the space (vacuum) in cylindrical coordinates (where r is the distance from origin on the xy plane, θ is the angle relative to the (+x) axis towards the (+y) axis on the xy plane).

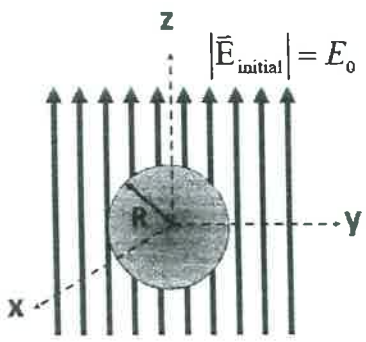

Figure 1

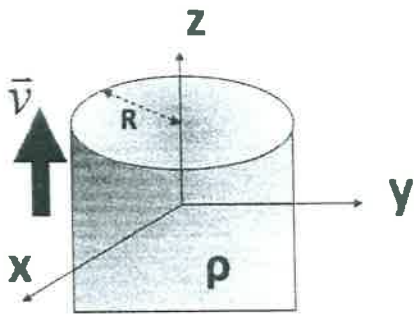

Figure 2

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4. (20%) Find the polarization (linear, circular, or elliptical and left-handed or right-handed) of the following fields:

(1) $\vec{E} = (j\hat{a}_x + \hat{a}_y)e^{-jkz}$ (4%)

(2) $\vec{E} = (\hat{a}_x - j\hat{a}_y)e^{+jkz}$ (4%)

(3) $\vec{E} = ((1+j)\hat{a}_y + (1-j)\hat{a}_z)e^{-jkx}$ (4%)

(4) $\vec{E} = ((2+j)\hat{a}_x + (3-j)\hat{a}_z)e^{-jky}$ (4%)

(5) $\vec{E} = (j\hat{a}_x + j2\hat{a}_y)e^{+jkz}$ (4%)

5. (10%) Prove that electromagnetic energy can not penetrate through a perfectly conducting surface.

6. (10%) A transmission line of characteristic impedance $Z_0 = 50(\Omega)$ is to be matched to a load impedance $Z_L = 30 + j10$ (Ω) through another transmission line with length ℓ and characteristic impedance Z_1. Please find out the required ℓ and Z_1.

7. (10%) Find the spacing d for a parallel-plate waveguide having a dielectric of $\varepsilon = 9\varepsilon_0$ and $\mu = \mu_0$such that 5000MHz is the cutoff frequency of the dominant mode.