

國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科目：統計學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

(Total 100 Points.) *There are 6 problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.*

1. An engineering system consisting of n components is said to be a k -out-of- n system ($k \leq n$) if the system functions if and only if at least k out of the n components function. Suppose that all components function independently of each other. (10 %)
 - (a) If the i th component functions with probability p_i , $i = 1, 2, 3, 4$, what is the probability that a 2-out-of-4 system function? (5 %)
 - (b) For a k -out-of- n system where each component functions with the same probability p , what is the probability that this k -out-of- n system function? (5 %)

2. The joint probability density function of X and Y is given by

$$f(x, y) = \frac{3}{16}xy^2, \quad 0 \leq x \leq 2, 0 \leq y \leq 2$$

(25 %)

- (a) Verify that this is indeed a joint density function. (5 %)
 - (b) Find $\mathbb{P}(Y > X)$. (5 %)
 - (c) Are X and Y independent? (5 %)
 - (d) Find the expected value of Y (i.e., μ_Y). (5 %)
 - (e) Find the variance of X (i.e., σ_X^2). (5 %)
3. If X_1 and X_2 are the amounts of time for performing two jobs in a production line, and X_1 and X_2 are independent exponential random variables with the same parameter β , where the probability density function is given by

$$f(x; \beta) = \frac{1}{\beta}e^{-\frac{x}{\beta}}, \quad x > 0$$

Let $Y_1 = X_1 + X_2$, which can be used to represent the total working time, and $Y_2 = X_1/(X_1 + X_2)$, which can be used to represent the proportion of the work time that will be performed by job one. (15 %)

- (a) Find the joint distribution of Y_1, Y_2 . (10 %)
- (b) Find the marginal distributions of Y_2 . (5 %)



國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科目：統計學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

4. We want to compare the cure rates (p_A and p_B) of two COVID-19 drugs, A and B. There are 600 and 800 patients treated by drugs A and B, respectively. Among that, we found that 360 and 440 patients were cured by drugs A and B, respectively. (20 %)
- (a) What is the 95% interval estimate of p_A ? (5 %)
- (b) Someone claims that the cure rate of drug A is greater than 0.65. Provide the hypotheses and test the hypotheses with $\alpha = 0.05$. (5 %)
- (c) If the true cure rate of drug A is 0.6, what are the type II error β and the power of the above test in (b)? (5 %)
- (d) Test if the cure rates of drug A and drug B are the same with $\alpha = 0.05$. (5 %)
5. NTUST gives each of its students a Statistics test. The scores on the test are normally distributed with mean μ and variance σ^2 . A simple random sample of 25 is taken with a sample mean of 75 and a sample standard deviation of 10. (20 %)
- (a) What is the 95% interval estimate of the mean μ ? (5 %)
- (b) What is the 95% interval estimate of the standard deviation σ ? (5 %)
- (c) Test if the average score μ is 80 at 95% confidence. (5 %)
- (d) Test if the standard deviation σ is less than 9 at 95% confidence. (5 %)
6. We want to study the relationship between the effectiveness (Y) and the dosage (X) of a certain COVID-19 drug and we postulate a model as follows. $Y_i = 20 + \beta_1 X_i + \varepsilon_i$ for $i = 1, 2, \dots, n$, where $E\{\varepsilon_i\} = 0$, $Var\{\varepsilon_i\} = \sigma^2$, and $Cov\{\varepsilon_i, \varepsilon_j\} = 0$ for all $i, j; i \neq j$. (10 %)
- (a) What is the appropriate least squares estimator b_1 of β_1 ? (5 %)
- (b) What is the variance of b_1 ? (5 %)



國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組
科目：統計學

Table A.3 (continued) Areas under the Normal Curve

Table with 13 columns (z from .00 to .09) and 20 rows (z from 0.0 to 3.4). Values range from 0.5000 to 0.9997.

Table A.3 Normal Probability Table

Table with 13 columns (z from .00 to .09) and 20 rows (z from -3.4 to -0.0). Values range from 0.0003 to 0.4641. Includes a normal distribution curve diagram.



國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科目：統計學

Appendix A Statistical Tables and Proofs

738

Table A.4 (continued) Critical Values of the t -Distribution

v	α									
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005			
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578			
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600			
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924			
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610			
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869			
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959			
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408			
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041			
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781			
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587			
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437			
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318			
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221			
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140			
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073			
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015			
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965			
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922			
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883			
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850			
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819			
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792			
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768			
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745			
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725			
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707			
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689			
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674			
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660			
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646			
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551			
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460			
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373			
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290			

737

Table A.4 Student t -Distribution Probability Table

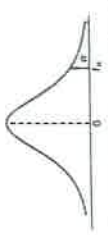
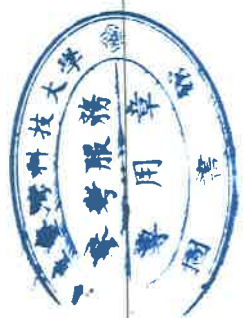


Table A.4 Critical Values of the t -Distribution

v	α									
	0.40	0.30	0.20	0.15	0.10	0.05	0.025			
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706			
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303			
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182			
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776			
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571			
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447			
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365			
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306			
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262			
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228			
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201			
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179			
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160			
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145			
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131			
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120			
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110			
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101			
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093			
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086			
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080			
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074			
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069			
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064			
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060			
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056			
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052			
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048			
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045			
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042			
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021			
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000			
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980			
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960			



國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科目：統計學

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

v	α										
	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266	
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466	
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515	
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.778	24.321	
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124	
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	
13	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527	
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124	
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.698	
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.791	
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819	
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314	
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796	
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619	
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051	
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475	
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892	
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301	
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702	
40	44.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403	
50	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660	
60	65.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608	

Table A.5 Chi-Squared Distribution Probability Table

v	α										
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50	
1	0.004393	0.0157	0.04628	0.0982	0.00393	0.0158	0.0642	0.102	0.148	0.455	
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386	
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366	
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357	
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351	
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346	
8	1.344	1.647	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	
13	3.565	4.107	4.765	4.909	5.892	7.041	8.634	9.299	9.926	12.340	
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339	
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337	
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	
27	11.808	12.878	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	
40	20.707	22.164	23.838	24.433	26.509	29.051	32.345	33.66	34.872	39.335	
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	
60	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	

