

## 國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組  
 科目：作業研究

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. Solve the following problem by the Branch-and-Bound method.

$$\text{Maximize } Z = 9x_1 + 4x_2 + 7x_3 + 2x_4$$

Subject to

$$15x_1 + 7x_2 + 12x_3 + 4x_4 + x_5 \leq 37 \quad (20\%)$$

$$x_1, x_2, x_3, x_4, x_5 \in \{0, 1\}$$

2. A library facing insufficient primary storage space for its book collection is considering the possibility of using secondary facilities to store portions of its collection. Each secondary storage facility  $j$  has a limited capacity  $b_j$  and an access cost  $v_j$  for retrieving a book from this facility. In addition, let  $a_i$  denote the number of books of a particular class  $i$  requiring storage and let  $u_i$  denote the expected rate (per unit time) that the books in this class need to be retrieved. The goal is to store the books in a way that minimizes the expected retrieval cost. (20%)
- (1) Show how to formulate the problem of determining an optimal policy as a minimum cost flow problem. (10%)
- (2) Show that the simple rule that repeatedly assigns items with the greatest retrieval rate to the storage facility with lowest access cost specifies an optimal solution of this library storage problem. (10%)
3. Four persons, one of whom has committed a terrible crime, made the following statements when questioned by the police.

Bill: "Grace lied."

Grace: "Bill did it."

Tiffany: "I didn't do it."

Jason: "Grace did it."

If only one of these four statements is false, construct a set of linear constraints to find the guilty person, using binary variables if necessary. (10%)



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4. We must invest all our money in two stocks:  $x$  and  $y$ . The variance of the annual return on one share of stock  $x$  is  $\text{var}(x)$ , and the variance of the annual return on one share of stock  $y$  is  $\text{var}(y)$ . Assume that the covariance between the annual return for one share of  $x$  and one share of  $y$  is  $\text{cov}(x, y)$ . If we invest  $a\%$  of our money in stock  $x$  and  $b\%$  in stock  $y$ , then the variance of our return is given by  $a^2\text{var}(x) + b^2\text{var}(y) + 2ab\text{cov}(x, y)$ . We want to minimize the variance of the return on our invested money. What percentage of the money should be invested in each stock? (15%)
5. Machine 1 is currently working. Machine 2 will be put in use at a time  $t$  from now. If the lifetime of machine  $i$  is exponential with rate  $\lambda_i$ ,  $i = 1, 2$ , what is the probability that machine 1 is the first machine to fail? (15%)
6. Consider the single queue model, where only one customer is allowed in the system. Customers who arrive and find the facility busy never return. Assume that the arrival distribution is Poisson with mean  $\lambda$  per unit time and that the service time is exponential with mean  $1/\mu$  time units. (20%)
  - (a) Set up the transition diagram, and determine the steady-state probabilities. (10%)
  - (b) Determine the average number in the system. (10%)

