國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：材料科學與工程系碩士班乙組

科　　目：工程數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. Solve the given differential equation. (15%)

$$(y^2 + xy^3)dx + (5y^2 - xy + y^3 \sin y)dy = 0$$

2. Solve the given differential equation. (15%)

$$x^2y'' + xy' - y = \frac{1}{x+1}$$

3. Find the inverse Laplace transform of the following functions. (10%)

(1) $\dfrac{3(s^2+4)}{s(s^2+4s+8)}$ (5%)

(2) $\dfrac{6}{(s^2+4s+13)^2}$ (5%)

4. Use the change of variable $t = -\frac{1}{2}\ln(1-x^2)$ on $0 \le x < 1$ to solve the given equation. (10%)

$$x(1-x^2)^2 y'' - (1-x^2)^2 y' + x^3 y = 0$$

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5. Solve Laplace's equation for a rectangular plate subject to the given boundary conditions. (20%)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

B.C.: $u(0, y)=0,\quad u(a, y)=0,\quad u(x, 0)=0,\quad u(x, b)=f(x)$

6. Are the following functions orthogonal to each other on the indicated interval. (10%)

$f_1(x) = e^x, f_2(x) = \sin x; [\pi/4, 5\pi/4]$

7. Find the gradient of the $f(x, y) = x^2 - 4y^2$ at the point (2, 4). (10%)

8. Find the eigenvalues and eigenvectors of the given matrix. (10%)

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$