

國立臺灣科技大學 111 學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科 目：工程數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. Solve the differential equations: (20%)
 - (a) $y'' - 16y = 2e^{4x}$ (10%)
 - (b) $y'' + y = 4x + 10\sin x$, $y(\pi) = 0$, $y'(\pi) = 2$. (10%)
2. The three vectors $\vec{v}_1 = [1 \ 1 \ 1 \ -1]^T$, $\vec{v}_2 = [2 \ -1 \ -1 \ 1]^T$, $\vec{v}_3 = [-1 \ 2 \ 2 \ 1]^T$. Convert them into an orthonormal vector set, where T denote the transpose of a vector. (10%)
3. Suppose that $F(s) = L\{f(t)\}$. Find the inverse Laplace transform of the function $F(s)$. (10%)

$$F(s) = \frac{s + 3}{(s + 2)(s^2 + 2s + 2)}$$

4. Check if the row vectors of the matrix A form a linearly independent set, and explain the reason. Then, please determine the eigenvalues, eigenvectors, and the dimension of the eigenspace of the matrix A . (20%)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

5. Expand $f(x) = e^{-x}$, $-\pi < x < \pi$, in complex Fourier series. (20%)
6. Solve $\nabla^2 T(x, y) = 0$, $0 < x < \infty$, $0 < y < 1$
 $T(0, y) = 1$, $T(\infty, y) = 0$, $\frac{\partial T(x, 0)}{\partial y} = 0$, $T(x, 1) = 0$ (20%)

