

國立臺灣科技大學 112 學年度碩士班招生試題

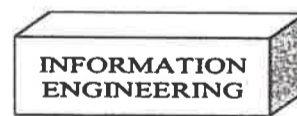
系所組別：資訊工程系碩士班
 科目：計算機數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. [15%] Please answer the following questions.
 - (1) (5%) Find the sequence with function $f(x) = e^{5x} + e^{-5x}$ as its exponential generating function.
 - (2) (10%) Use generating functions to solve the recurrence relation $a_n = 6a_{n-1}$ with the initial condition $a_0 = 7$.
2. [10%] Construct a finite-state machine that gives an output of 1 if the number of input symbols read so far is divisible by 3 and an output of 0 otherwise. (0 and 1 are the only two input symbols. s_k denotes the k th state. An arrow with the input and output pair represents each transition. Be sure to indicate the initial state.)
3. [10%] Which of the following statement(s) is/are correct?
 - A. Each of the following statements can be used to express the fact that there is a unique element x such that $P(x)$ is true.
 $\exists x P(x) \wedge \forall x \forall y (P(x) \wedge P(y) \rightarrow x = y)$
 $\exists x (P(x) \wedge \forall y (P(y) \rightarrow x = y))$
 $\exists x \forall y (P(y) \leftrightarrow x = y)$
 - B. The set of irrational numbers is countable.
 - C. If 251 distinct integers are randomly generated between 500 and 1000 (including 500 and 1000), we can always find two selected ones with the sum of 1500.
 - D. If three events are pairwise independent, they are also mutually independent.
 - E. Two trees are isomorphic if and only if they have the same number of leaf nodes.
4. [15%] Suppose that the letters (space is not included) of "COMPUTER SCIENCE" and "INFORMATION ENGINEERING" are put into BOX 1 and BOX 2, respectively.



BOX 1 (totally 15 letters)



BOX 2 (totally 22 letters)

- (1) (5%) I select one of these two boxes at random and pick a letter from that box at random. If I get letter "E", what is the probability that this letter is from the BOX 1? Round your answer off to two decimal places.
- (2) (5%) Among 7 letters in the word "SCIENCE", 6 of them are selected to form a string. How many different strings can be made?
- (3) (5%) Find the number of solutions to $x + y + z = 22$, where x, y , and z are positive even integers, and assume that $x > 2, y > 4$, and $z < 10$. What is the number of solutions if the summation becomes 48? In other words, $x + y + z = 48$ without changing any other constraint.



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5. [10%] Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$,

$$T\left(\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -5 \end{bmatrix}, \text{ and } T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

(1) (5%) Find $T\left(\begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}\right)$.

(2) (5%) Find the standard matrix of T .

6. [5%] Let $u_1 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 7 \\ 5 \\ 4 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$, $u_4 = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$, and the matrix $A =$

$2u_1u_2^T + u_3u_4^T$. Find the rank of A .

7. [10%] Let $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ and $P = \begin{bmatrix} \cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) \end{bmatrix}$.

(1) (5%) Find the eigenvalue λ_M of A that has the largest magnitude.

(2) (5%) Let $B = PAP^T$, determine whether B is diagonalizable.



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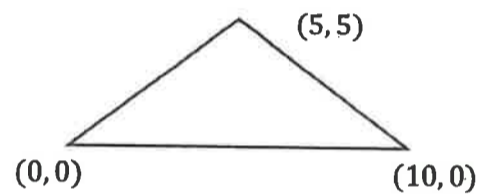
8. [10%] We have two distinct vectors x_1, x_2 and they satisfy linear systems $Ax = b$ and $Cx = d$, respectively, where A, C are $n \times n$ matrices. We also know that the $n \times (n+1)$ matrix $[C; d]$ can be obtained by a series of three row operations from the $n \times (n+1)$ matrix $[A; b]$. What you can conclude about the matrix A ?

9. [15%] We have a linear transformation $T(x) = Ax$ where

$$A = \begin{bmatrix} 1 & 2 \\ 11/4 & 1/2 \end{bmatrix}.$$

Answer the following two questions.

- (1) (10%) For a triangle R given below, compute the area of the region $T(R)$.



- (2) (5%) Following (a), now if the triangle top $(5,5)$ moves to the right to become $(7,5)$ and $T(x) = BAx$, where

$$B = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, \text{ for } \theta = 90^\circ,$$

compute the area of $T(T(R))$.

