（ 總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分 ）

1. (25%) Solve the following problem by the revised simplex method.

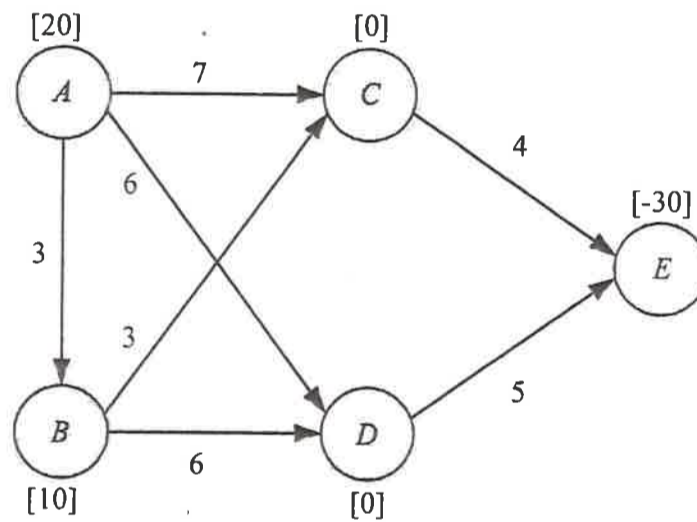$$\text{Maximize} \quad Z = x_1 - x_2 + 2x_3$$
$$\text{subject to} \quad x_1 + x_2 + 3x_3 \le 6$$
$$-x_1 + x_2 + x_3 \le 2$$
$$2x_1 - x_2 + x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

2. (25%) Consider the minimum cost flow problem shown below, where the new flow values are given by the nodes, and the costs per unit flow are given by the arcs. The arc capacities of (A, C) and (B, C) are 20 and 15, respectively.

Starting from the initial basic feasible solution with basic arcs (A, D), (B, C), (C, E) and (D, E), use the network simplex method to solve the problem.

國立臺灣科技大學 112 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組
科　　目：作業研究

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

3. **(15%)** Consider the following function:

$$f(x) = 5x_1 + 2x_2^2 + x_3^2 - 3x_3x_4 + 4x_4^2 + 2x_5^4 + x_5^2 + 3x_5x_6 + 6x_6^2 + 3x_6x_7 + x_7^2$$

Show that $f(x)$ is convex by expressing it as a sum of functions of one or two variables and then showing that all these functions are convex.

4. **(15%)** For each of the following chains, determine whether the Markov chain is ergodic. Also, for each chain, determine the recurrent, transient, and absorbing states.

$$P_1 = \begin{bmatrix} 0 & .8 & .2 \\ .3 & .7 & 0 \\ .4 & .5 & .1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} .2 & .8 & 0 & 0 \\ 0 & 0 & .9 & .1 \\ .4 & .5 & .1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. **(20%)** A maintenance person has the job of keeping two machines in working order. The amount of time that a machine works before breaking down has an exponential distribution with a mean of 10 hours. The time then spent by the maintenance person to repair the machine has an exponential distribution with a mean of 8 hours.

(1) **(5%)** Show that this process fits the birth-and-death process by defining the states, specifying the values of birth and death rates (λ_n, μ_n), and then constructing the rate diagram.

(2) **(5%)** Calculate the steady-state probabilities.

(3) **(5%)** Determine the proportion of time that the maintenance person is busy.

(4) **(5%)** Determine the proportion of time that any given machine is working.