

國立臺灣科技大學 112 學年度碩士班招生試題
系所組別：材料科學與工程系碩士班乙組
科目：工程數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. (10%) Solve the given differential equation.

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$$

- (1) (2%) Is the first order ODE exact?
(2) (8%) Find the solution

2. (10%) Solve the given differential equation.

$$y'' + 4y = \frac{1}{2} - \frac{\cos 2x}{2}$$

- (1) (4 %) Find Homogeneous Solution
(2) (6%) Use undetermined coefficient to find Particular Solution and write down the complete solution

3. (15%) Find the solution of the second order linear differential equation

$$x^2 y'' - 2xy' + 2y = x^2 + 2$$

4. (15%) Use Laplace Transform to solve the ODE

$$y' + 4y = f(t), \quad y(0) = 0$$

$$f(t) = \begin{cases} t-1 & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (1) (5%) Find out Laplace transform of $f(t)$
(2) (10%) Find the solution



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5. (10%) Determine whether the vectors $\mathbf{a} = 4\mathbf{i} + 6\mathbf{j}$, $\mathbf{b} = -2\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$, and $\mathbf{c} = 5/2\mathbf{i} + 3\mathbf{j} + 1/2\mathbf{k}$ are coplanar.

6. (15%) If $\mathbf{A} = \begin{pmatrix} 4 \\ 8 \\ -10 \end{pmatrix}$ and $\mathbf{B} = (2 \ 4 \ 5)$,

find (1) (5%) $\mathbf{A}^T\mathbf{A}$

(2) (5%) $\mathbf{B}^T\mathbf{B}$

(3) (5%) $\mathbf{A} + \mathbf{B}^T$

7. (10%) Find the volume of the parallelepiped for which the given vectors are three edges.

$\mathbf{a} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $\mathbf{c} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$

8. (15%) Find the eigenvalues and eigenvectors of the given matrix.

$$\begin{bmatrix} 2 & -1 & 0 \\ 5 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

