國立臺灣科技大學
113學年度碩士班招生

試題

系所組別：0110工業管理系碩士班甲組

科　　目：作業研究

<<501103>>

國立臺灣科技大學 113 學年度碩士班招生試題
系所組別：工業管理系碩士班甲組
科　　目：作業研究

（總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分）

1. **(30%)** Consider the following problem

$$
\begin{aligned}
\text{Maximize} \quad & Z = 2x_1 - x_2 + x_3 \\
\text{subject to} \quad & 3x_1 - 2x_2 + 2x_3 \le 15 \\
& -x_1 + x_2 + x_3 \le 3 \\
& x_1 - x_2 + x_3 \le 4 \\
& x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0
\end{aligned}
$$

Please find the optimal solution and also conduct sensitivity analysis by independently investigating each of the following changes in the original model. If the test of sensitivity analysis fails, please find a new optimal solution.

(a) **(5%)** Find the optimal solution of this problem

(b) **(5%)** Change the right-hand sides to $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 2 \end{bmatrix}$

(c) **(5%)** Change the coefficients of x_1 and x_2 to $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} c_2 \\ a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 2 \\ 3 \end{bmatrix}$, respectively

(d) **(5%)** Change the objective function to $Z = 5x_1 + x_2 + 3x_3$

(e) **(5%)** Change constraint 1 to $2x_1 - x_2 + 4x_3 \le 12$

(f) **(5%)** Introduce a new constraint $2x_1 + x_2 + 3x_3 \le 60$

2. **(20%)** A contractor has to haul gravel to three building sites. The contractor can purchase as much as 18 tons at a gravel pit in the north of the city and 14 tons at one in the south. The contractor needs 10, 5, 10 tons at sites 1, 2, and 3, respectively. The purchase price per ton at each gravel pit and the hauling cost per ton are given as the following Table

Pit	Hauling cost per Ton at site			Price per ton
	1	2	3	
North	$100	$190	$160	$300
South	$180	$110	$140	$420

(a) **(5%)**Formulate a linear programming model for this problem

(b) **(10%)** Find an optimal solution for this problem

國立臺灣科技大學113學年度碩士班招生試題
系所組別：工業管理系碩士班甲組
科　　目：作業研究

（總分為100分；所有試題務必於答案卷內頁依序作答，否則不予計分）

(c) **(5%)** Given the optimal solution and the value of c_{ij} for each basic variable x_{ij} is fixed at the value given in the Table. Please determine the allowable range for each of value of c_{ij} for the non-basic variable x_{ij} and explain how the information is useful to the contractor.

3. **(20%)** Given the following transition matrix with states 1 and 2 as absorbing states, what is the probability that units in states 3 and 4 end up in each of the absorbing states?

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.2 & 0.1 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix} \end{array}$$

4. **(15%)** Suppose you arrive at a single-teller bank to find five other customers in the bank, one being served and the other four waiting in line. You join the end of the line. If the service times are all exponential with rate μ, what is the expected amount of time you will spend in the bank?

5. **(15%)** An average of 10 jobs per hour arrive at a job shop. Interarrival times of jobs are exponentially distributed. It takes an average of 10/3 minutes (exponentially distributed) to complete a job. Unfortunately, 1/3 of all completed jobs need to be reworked. Thus, with probability 1/3, a completed job must wait in line to be reworked. In the steady state, how many jobs would be in the job shop? What would the answer be if it took an average of 5 minutes to finish a job?