國立臺灣科技大學

115學年度碩士班招生

試題

系所組別：1500資訊工程系碩士班

科　　　目：計算機數學

<<515002>>

（總分為100分；所有試題務必於答案卷內頁依序作答）

1. (15%) Please answer the following questions about graph algorithms and properties.

(a) (5%) What is the time complexity of the following graph algorithm?

Input: Graph G with n nodes, and an integer constant k
Output: Success/Failure

```
for each subset S of k nodes
    if S is an independent set then
        return Success
    end if
end for
return Failure
```

(b) (5%) What is the time complexity of the following graph algorithm?

Input: Graph G with n nodes
Output: Set S*

```
S* ← φ
for each subset S of nodes
    if S is an independent set then
        if |S| > |S*| then
            S* ← S
        end if
    end if
end for
return S*
```
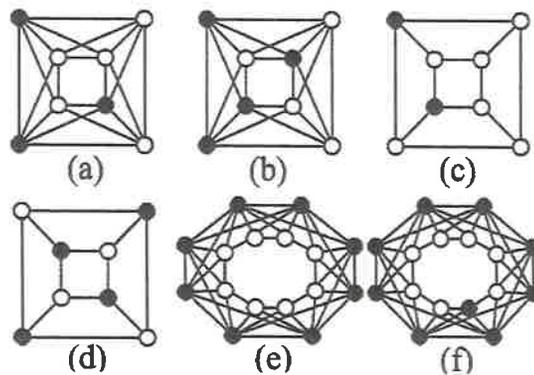
(c) (5%) Among the graphs as shown in figures (a) to (f), which one(s) are *bipartite*?

(a)　(b)　(c)
(d)　(e)　(f)

（總分為100分；所有試題務必於答案卷內頁依序作答）

2. (10%) You are given a knapsack with a maximum weight capacity of 20kg and a maximum volume capacity of 20L. There are 4 items available, each with a specific weight (w), volume (vol), and value (val).

The items are defined as follows:

- Item 1: w=10, vol=8, val=16
- Item 2: w=8, vol=11, val=15
- Item 3: w=12, vol=9, val=22
- Item 4: w=6, vol=6, val=9

Let $OPT(i, w, v)$ denote the maximum value obtainable by selecting from the first i items such that the total weight does not exceed w and the total volume does not exceed v.

Please calculate the value of $OPT(4, 20, 20)$, and list the items in the optimal selection.

3. (12%) Ten books, consisting of six distinct magazines (labeled A, B, C, D, E, and F) and four identical textbooks, are arranged on a shelf.

(a) (6%) In how many arrangements does Magazine A appear somewhere between Magazine B and Magazine C?

(b) (6%) Twenty identical stickers are distributed to magazines A, B, and C such that no two magazines receive the same number of stickers. How many distributions are possible?

4. (13%) Three fair six-sided dice are rolled. Define the random variable X as follows:
$$X = \begin{cases} 1, & \text{if the product of the three numbers is even.} \\ 0, & \text{if the product of the three numbers is odd.} \end{cases}$$

(a) (5%) Describe the sample space S of this problem in a compact form. Note that enumeration will receive only partial credit.

(b) (8%) Find the distribution of X.

5. (10%) Show that if matrices A and B are similar, then they have the same determinant, i.e., $det(A) = det(B)$.

（總分為100分；所有試題務必於答案卷內頁依序作答）

6. (15%)

(a) (5%) Given the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$, determine its eigenvalues and the corresponding eigenvectors.

(b) (10%) Consider the general matrix $\begin{bmatrix} a & a+1 \\ 1 & 0 \end{bmatrix}$, where a is a real number. Formulate a conjecture about its eigenvalues and eigenvectors, and verify your conjecture.

7. (15%) Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $(1,2,1)$ and $(-2,1,0)$ and p be the vector $(1,3,1)$.

(a) (10%) Find the 3×3 projection matrix onto W.

(b) (5 %) Find the orthogonal projection of the vector p onto W.

8. (10%) Suppose you are predicting the CPU temperature T based on the clock speed C and the voltage V. You have collected the following set of observations:

Clock Speed C (GHz)	Voltage V (V)	Temperature T (°C)
1	1	40
2	1	50
1	2	80

Find the best linear model to make the prediction.