國立臺灣科技大學

115學年度碩士班招生

試題

系所組別：1200自動化及控制研究所碩士班

科　　目：工程數學

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(總分為100分;所有試題務必於答案卷內頁依序作答)

1. **(10%)** Find the general solution of the following second-order non-homogeneous linear differential equation with variable coefficients:

$$x^2y''-3xy'+3y=2x^4e^x,\quad x>0$$

2. **(10%)** Solve the following integral-differential equation for $y(t)$:

$$y'(t)=\cos t+\int_0^t y(\tau)\cos(t-\tau)d\tau,\quad y(0)=1$$

3. **(10%)** Let $\mathbf{F}=(y+\sin x)\mathbf{i}+(z^2+\cos y)\mathbf{j}+x^3\mathbf{k}$. Let C be the boundary of the triangle formed by the intersection of the plane $x+y+z=1$ and the coordinate planes $x=0$, $y=0$, $z=0$ in the first octant. The curve c is oriented counter-clockwise when viewed from above. Use **Stokes' Theorem** to evaluate the line integral:

$$\oint_c \mathbf{F}\cdot d\boldsymbol{r}$$

4. **(10%)** Consider the differential equation: $2x^2y''+3xy'-(x^2+1)y=0$.

   (1) **(5%)** Find the indicial equation and its roots.

   (2) **(5%)** Determine the recurrence relation for the coefficients *an* corresponding to the larger root.

5. **(10%)** Given $\mathbf{F}=(x^3+e^{yz})\mathbf{i}+(y^3+\sin(xz))\mathbf{j}+(z^3+\ln(x^2+y^2+1))\mathbf{k}$. Use the **Divergence Theorem** to calculate the total outward flux $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ across the closed surface S: $x^2+y^2+z^2=a^2$.

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6. **(20%)** Consider a 3x3 matrix $\mathbf{A}=\begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$

(1) **(8%)** Find Eigenvalues and the corresponding Eigenvectors of **A**.

(2) **(6%)** Solve the system $X' = \mathbf{A}X$.

(3) **(6%)** Diagonalize the matrix **A**.

7. **(15%)** Let $f(x) = \begin{cases} 0 & \text{for } -\pi \leq x \leq 0 \\ x & \text{for } 0 < x \leq \pi \end{cases}$

Write the Fourier series of $f(x)$ on $[-\pi, \pi]$ and obtain a trigonometric series expansion of

$\int_{-\pi}^{x} f(t)dt$ on $[-\pi, \pi]$

8. **(15%)** The trajectory of a flying object is a three-dimensional curve , and its parametric equation is:

$x(t)=\sqrt{t}\,sin(\pi t/6)$, $y(t)=\sqrt{t}\cos(\pi t/6)$, $z(t)=\sqrt{t}$; unit: m for $0 \leqq t \leqq 36$ seconds.

(1)**(10%)** Find the position vector $\boldsymbol{p}(t)$= x**i**+y**j**+z**k**, velocity vector $\boldsymbol{v}(t)$, and speed value $u(t)$, $u(3)$, $u(6)$, $u(9)$.

(2)**(5%)** Draw the approximate path for t = 0, 2, 4,..12 seconds on the x-y plane (top view).