

(命題用紙)

八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所 別：工程技術研究所

學程別：自動化及控制學程

組別：甲組、丙組

科目：工程數學

1. Find the general solution of the following equations.

$$(1) \quad \frac{dy}{dx} = \frac{y^2 - 2xe^x}{2y} \quad (10\%)$$

$$(2) \quad y'' + 4y' + 4y = \frac{e^{-2x}}{x^2} \quad (10\%)$$

$$(3) \quad \frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = f(t) \quad (10\%)$$

$$\text{where } f(t) = 1 - \sinh t + \int_0^t (1 + \tau)f(t - \tau)d\tau$$

$$(4) \quad (1 - x^2)y'' - 2xy' + 2y = 0 ; \quad (10\%)$$

One solution $y = x$ is known.

2. A 3×3 symmetric matrix A has eigenvalues 6, 3 and 1. The eigenvectors corresponding to the eigenvalues 6 and 1 are $[1 \ 2 \ 0]^T$ and $[-2 \ 1 \ 0]^T$ respectively. Find the eigenvector corresponding to the eigenvalue 3, and hence determine the matrix A . (10%)

3. Evaluate $\iint xy dx dy$ over the region in $x \geq 0, y \geq 0$ bounded by $y = x^2 + 4, y = x^2, y = 6 - x^2$ and $y = 12 - x^2$. (10%)

4. Given

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

Calculate A^r , where r is an integer greater than 2. (10%)

5. Obtain the Laplace transform of the piecewise-continuous causal function

$$f(t) = \begin{cases} 2t^2 & (0 \leq t < 3) \\ t + 4 & (3 \leq t < 5) \\ 9 & (t \geq 5) \end{cases} \quad (10\%)$$



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6. The temperature distribution in a thin circular disk whose upper and lower faces are insulated, if independent of θ , satisfies the equation

$$\frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) = c^2 r \frac{\partial u}{\partial t} \quad 0 \leq r \leq 1$$

with initial condition

$$u(r, 0) = 100$$

and boundary condition

$$u(1, t) = 0$$

- Find an equation whose roots are the eigenvalues of this problem.
- Assume that λ_n ($n=1, 2, 3, \dots$) are the eigenvalues of this problem. Find the eigenfunctions ϕ_n corresponding to eigenvalues λ_n .
- Show the orthogonality of eigenfunctions.
- Find the solution $u(r, t)$ of this problem.

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