

所 別：管理技術研究所

學程別：工業管理學程

組別：

科目：微積分

(1) 試求 $\int_0^{\infty} e^{-x^2} dx$ 之值。(16%)(2) 試求 $\lim_{x \rightarrow 0} (\csc^2 x - \frac{1}{x^2})$ 之值。(16%)(3) Find the area of the region outside the cardioid $r = 1 + \cos \theta$ and inside the circle $r = \sqrt{3} \sin \theta$. (16%)(4) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy + y^3}{x^2 + y^2}$ does not exist. (12%)

(5) 下列敘述，正確者，請證明之，錯誤者，請舉反例。(24%)

(a) if a series $\sum_{n=1}^{\infty} a_n$ diverges, then its sequence of partial sums is unbounded.(b) if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.(c) if $\int_a^b f(x) dx = 0$, then $f(x) = 0$ for all x in $[a, b]$.(d) $\int_0^1 \int_0^x f(x, y) dy dx = \int_0^1 \int_0^y f(x, y) dx dy$.(6) Let $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$ (16%)(a) Find $f'(0)$ from the definition of the derivative.(b) Show that $f'(x)$ is discontinuous at $x=0$.