(命題用紙)

八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所　別：管理技術研究所　　科目：統計學

學程別：企業管理學程　　組別：

1. By randomly inquiring five students in a class, the teacher found that only one has working experiences. Use 90% confidence level, and determine the maximum proportion of students in the class who have ever worked before.**(15%)**

2. In order to test the assumption of a Poisson distribution for the number of arrivals during weekday morning hours, a store employee randomly selects a sample of 100 5-minute intervals during weekday mornings over a 3-week period. For each 5-minute intervals in the sample, the store employee records the number of customer arrivals. In summarizing the data, the employee determines the number of 5-minute interval having no arrivals, the number of 5-minute intervals having one arrival, the number of 5-minute intervals having two arrivals, and so on. These data are summarized in the following table.

Number of Customers Arriving	Observed Frequency
0	5
1	15
2	21
3	25
4	18
5	10
6	6

Use $\alpha = 0.05$ and test whether the number of customer arrivals follows a Poisson distribution.**(15%)**

3. Consider the simple regression model

$$Y_i = \alpha + \beta X_i + \varepsilon_i \quad ; \quad i = 1,2,\cdots,n$$

The estimated regression equation is

$$\hat{Y}_i = a + bX_i \quad ; \quad i = 1,2,\cdots,n$$

And the residual is

$$e_i = Y_i - \hat{Y}_i \quad ; \quad i = 1,2,\cdots,n$$

The coefficient of determination is

$$R^2 = \frac{SSR}{SST}$$

where $SST = \sum_{i=1}^{n} (Y_i - \overline{Y})^2$ is the total sum of squares, and

$SSR = \sum_{i=1}^{n} (\hat{Y}_i - \overline{Y})^2$ is the sum of squares due to regression.

Please show that

a. R is equal to the correlation coefficient between Y and $\hat{Y}$, i.e. $\rho_{Y,\hat{Y}}$.**(10%)**

b. R is equal to the correlation coefficient between Y and X, i.e. $\rho_{Y,X}$.**(10%)**

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4. (30%) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel which returns him to his cell after two-day's travel. The second leads to a tunnel which returns him to his cell after three-day's travel. The third door leads immediately to freedom.

(a) Assuming that the prisoner will always select doors, 1, 2, and 3 with probabilities 0.5, 0.3, 0.2, what is the expected number of days until he reaches freedom?

(b) Assuming that the prisoner is always equally likely to choose among those doors that he has not used, what is the expected number of days until he reaches freedom?

(c) For parts (a) and (b), find the variance of the number of days until our prisoner reaches freedom

5. (20%) A sample of size 1000 adult women is drawn for a survey in which each woman is asked two questions:

(i) How many children did your mother have?

(ii) How many children did you have?(Assume their childbearing days are over.)

Suppose the distribution of number of children per woman has not changed over time and is $\alpha_j = \frac{1}{3}, j = 1,2,3$. If we did not know that $\{\alpha_j\}$ has not changed, the results of the average response to question (i) and the average response to question (ii) might lead us to the conclusion that family sizes are decreasing. Instead, what is the explanation for these results? Would increasing the sample size help? Why?