八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所　別：管理技術研究所

學程別：資訊管理學程　組別：　科目：統計學

STATISTICS　April, 1996

1.(3% each)

(a) Define the *random variable.* Give one example.

(b) Define the *sampling distribution* of a *statistic.* Give one example.

(c) Why do we need to learn *probability* before *statistics.*

(d) When estimating θ by the estimator $\hat{\theta}$, why is it important to report $\widehat{\mathrm{Var}}[\hat{\theta}]$, which estimates the variance of $\hat{\theta}$, in addition to reporting $\hat{\theta}$. Support your answer.

(e) We usually use the sample-mean difference, $\bar{X} - \bar{Y}$, to estimate the population-mean difference, $\mu_X - \mu_Y$, and use the sample-variance ratio, S_X^2/S_Y^2, to estimate the population-variance ratio, σ_X^2/σ_Y^2. Why do we not estimate μ_X/μ_Y by $\bar{X}/\bar{Y}$ and estimate $\sigma_X^2 - \sigma_Y^2$ by $S_X^2 - S_Y^2$?

(f) There are 20 females and 30 males in a class. Randomly select one person and find that the selected person's weight is 45 kilograms. What is the probability, compared to 20/50, that the selected person is a female? Support your answer.

2.(8%) Suppose that there are k defectives in a lot of N $(N > k)$ electrical components. The manager randomly inspects component one by one from the lot until he finds the first defective component. What is the distribution of the number of components inspected

(a) if the manager does not replace the component after inspection,

(b) if the manager replaces the good component inspected with a defective component after each inspection.

3.(8%) Particle A moves randomly along x-axis from the origin to point $(\alpha, 0)$ and is located at $(U, 0)$, where U is a continuous random variable with density $f_U(u) = 1/\alpha, 0 < u < \alpha$. Particle B moves randomly along y-axis from the origin to point $(0, \beta)$ and is located at $(0, V)$, where V is a continuous random variable with density $f_V(v) = 1/\beta, 0 < v < \beta$. Find the probability that the distance between particles A and B is smaller than α.

4.(8%) There are $k > 2$ different doors to enter and leave the room. Suppose that two people A and B simultaneously enter and then simultaneously leave the room. Any one is not allowed to enter and leave through the same door. How many ways are there for A and B to enter and then leave the room if they are not allowed to use the same door simultaneously?

5.(8%) A chemical experiment is performed under pressure x_i, which is deterministic, and yield Y_i is observed. Suppose a random sample $\{(x_i, Y_i), i = 1, 2, \ldots, n\}$ is collected. Estimate by least-squared-errors method the parameters α, β, and γ in the following models.

(a)$\mathrm{E}(Y_i) = \alpha + \beta x_i^2$.

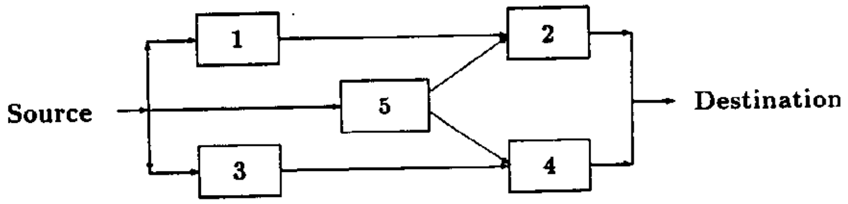
(b)$\mathrm{E}(Y_i) = \gamma$.

八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所　別：管理技術研究所

學程別：資訊管理學程　　組別：　　科目：統計學

6.(8%) Consider the following electrical circuit, where each switch functions independently with probability p of being operable. What is the probability that the electric current can pass from source to destination?



7.(10%) Let X be a binomial random variable with probability function $b(x;n,p) = n!/[x!(n-x)!]p^x(1-p)^{n-x}$. When $n \to \infty, p \to 0$, and $\lambda = np$ remains constant, show that $b(x;n,p) \to p(x;\lambda)$, where $p(x;\lambda) = e^{-\lambda}\lambda^x/x!$ is the poisson probability function with parameter λ.
(Hint: $\lim_{n\to\infty}(1-\lambda/n)^n = e^{-\lambda}$)

8.(10%) The speed of a molecule in a uniform gas at equilibrium is a random variable V whose probability function is given by $f(v) = kv^2e^{-bv^2}$, if $v > 0$, and 0, otherwise, where k is an appropriate constant and parameter b depends on the absolute temperature and mass, m, of the molecule. Find the probability function of the kinetic energy, $W = mV^2/2$, of the molecule.

9.(4% each) The life of a certain device has a constant failure rate 0.01 failures/hour.
(a) What is the mean time to failure.
(b) What is the probability that 200 hours will pass before observing a failure?
(c) What is the probability of observing 2 failures within 100 hours?

10.(5% each) Suppose that $\{Y_1, Y_2, Y_3\}$ denotes a random sample from an exponential distribution with mean θ. Consider the following five estimators of θ:

$$\hat{\theta}_1 = Y_1 \qquad \hat{\theta}_2 = \frac{Y_1+Y_2}{2} \qquad \hat{\theta}_3 = \frac{Y_1+2Y_2}{3}$$

$$\hat{\theta}_4 = \min\{Y_1, Y_2, Y_3\} \qquad \hat{\theta}_5 = \frac{Y_1+Y_2+Y_3}{3}$$

(a) Which estimators are unbiased?
(b) Among the unbiased estimators, which one has the smallest variance?