

八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所別：化學工程技術研究所

組別：

科目：工程數學

1. (10%) Use Gaussian elimination to solve the following linear system

$$\begin{aligned} 2x_1 + 6x_2 + x_3 &= 7 \\ x_1 + 2x_2 - x_3 &= -1 \\ 5x_1 + 7x_2 - 4x_3 &= 9 \end{aligned}$$

2. (15%) Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

3. (10%) Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at $(1,-1,2)$ in the direction of $6i + 2j + 3k$.

4. (15%) Expand $f(x) = x^2, 0 < x < L$ (a). in a cosine series (b). in a Fourier series.

5. (10%) Find the general solution of the ordinary differential equation.

$$9x^2y'' - 27xy' + (9x^2 + 35)y = 0$$

Hint: i) use $u = yx^{-2}$ to transform the differential equation into a Bessel equation.
ii) write the general solution of the Bessel equation, and then write the general solution of the original equation.

6. (10%) Solve the system of ordinary differential equations using the Laplace transform.

$$x'' - 2x' + 3y' + 2y = 4 + 2\delta(t)$$

$$2y' + 3y - x' = 0$$

$$\text{IC: } x(0) = y(0) = 0, \quad x'(0) = -2$$

7. (10%) Solve the initial value problem

$$x^2y'' + 7xy' + 9y = 27\ln(x); \quad \text{with IC. } y(1) = 1, \quad y'(1) = -4$$

8. (20%) Solve the boundary value problem

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} + 2 = 0 \quad (0 < x < \ell, 0 < y < \ell)$$

$$\text{BC 1. } \frac{\partial T(0,y)}{\partial x} = 0$$

$$\text{BC 2. } \frac{\partial T(x,0)}{\partial y} = 0$$

$$\text{BC 3. } T(\ell, y) = T_\infty$$

$$\text{BC 4. } T(x, \ell) = T_\infty$$

