八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所別：電機工程技術研究所　　組別：電力組、控制組　　科目：工程數學

1. Solve the integral equation (10%)

$$y(t) = t + \int_0^t y(\tau)\sin(t-\tau)d\tau$$

2. Solve the initial value problem (10%)

$$y'' - 4y' + 3y = 10e^{-2x}, \quad y(0) = 1, \quad y'(0) = -3.$$

3. Solve the initial value problem (10%)

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0, \quad y(1) = 2, \quad y'(1) = 1, \quad y''(1) = -4.$$

on any open interval I on the positive x-axis containing $x=1$.

4. Evaluate the following integral

(a). $\oint_c \frac{e^{2z}}{(z-1)^3}dz$, C is any simple closed path enclosing 1. (10%)

(b). $\int_{-\infty}^{\infty}\frac{dx}{x^4+1}$, (10%)

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5. Let $A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix}$.

   a. (5%) Find the null space and the column space of A,

   b. (8%) find the eigenvalues and the corresponding eigenvectors for A,

   c. (7%) find a matrix Q such that $Q^{-1}AQ$ is a Jordan canonical form.

6. (10%) Under the traditional definition of inner product, please find the closest vector to the vector $y = [1, -3, 4, 2, 8]$ in the vector space spanned by $x_1 = [1, 2, -3, 1, 0]$ and $x_2 = [0, 1, 3, 3, 1]$.

7. The probability distribution of random variables, X and Y, is shown as follows.

	X=1	X=2	X=3	X=4	X=5	X=6
Y=1	0.02	0.02	0.04	0.02	0.02	0.01
Y=2	0.02	0.04	0.06	0.04	0.02	0.01
Y=3	0.04	0.06	0.09	0.06	0.04	0.01
Y=4	0.02	0.04	0.06	0.04	0.02	0.01
Y=5	0.02	0.02	0.04	0.02	0.02	0.01
Y=6	0.01	0.01	0.01	0.01	0.01	c

   a. (5%) Please find the constant c,

   b. (5%) compute the variance for X,

   c. (5%) compute the expectation for X given $Y < 3$, and

   d. (5%) evaluate the correlation between X and Y.