

1. (20%)

Given a system described by:

$$\frac{d y(t)}{d t} + 2y(t) = x(t) \quad y(0) = 0$$

Consider the following two inputs:

$$x_1(t) = 0 \quad \text{for all } t$$

$$x_2(t) = \begin{cases} 0, & t < -1 \\ 1, & t > -1 \end{cases}$$

Let $y_1(t)$ be the response to $x_1(t)$ and $y_2(t)$ be the response to $x_2(t)$.

(a) Find $y_1(t)$ and $y_2(t)$.

(b) Is this system causal? Explain your answer or *no partial credit will be given*.

(c) If we assume that the **initial rest** condition is applied to the above system, what are the new answers for part (a) and (b)?

2. (10%)

Find the initial and final values of the time domain representation $f(t)$ corresponding to the function:

$$F(s) = \frac{2s+1}{s(s+1)}$$

(a) Use the initial value and final value properties of the unilateral Laplace transform.

(b) Verify your results by inverting $F(s)$.

3. (10%)

Compute the Fourier transform of each of the following signals:

(a) $[t e^{-2t} \sin 4t] u(t)$ where $u(t)$ is the unit step function.

(b) $\sin t + \cos\left(2\pi t + \frac{\pi}{4}\right)$



4. (20%)

Consider the two-mass mechanical system shown in Figure P4.

- (a) Find the electrical circuit that is a force-current analog of system in Figure P4.
- (b) Find the transfer function $Y(s)/U(s)$.

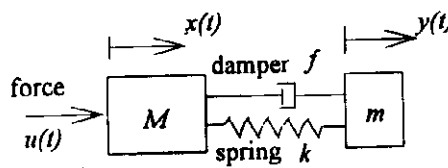


Figure P4. A two-mass mechanical system.

5. (20%)

A closed-loop system with unit feedback has a transfer function

$$T(s) = \frac{10(s+1)}{s^2 + 9s + 10}$$

- (a) Determine the open-loop transfer function $G(s)$.
- (b) Is the open-loop system stable? Is the closed-loop system stable?
- (c) Draw the log-magnitude-phase plot (i.e. $20 \log|G(j\omega)|$ vs. $\angle G(j\omega)$) of $G(s)$.
- (d) Determine the gain margin and the phase margin.

6. (20%)

The model for a DC motor position control system is shown in Figure P6. The goal is to select K_1 and K_2 so that the peak time is 0.8 seconds and the overshoot is about 1% for a step input.

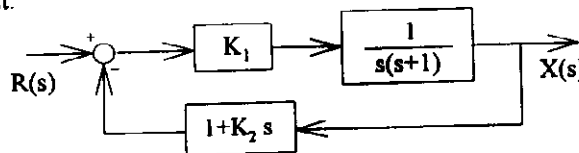


Figure P6. Position control of a DC motor.

