八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所別：電機工程技術研究所　　組別：計算機組　　科目：離散數學

A. 單一選擇題：

以下第 1 至 14 題，每題答對者得 5 分，答錯每題倒扣 1 分，未答者以零分計算；倒扣至本 A 部分零分為止。

( ) 1. A lot containing seven components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.
(a) 3.5 , (b) 2.3 , (c) 1.7 , (d) 0.6 .

( ) 2. One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?
(a) 17/48 , (b) 13/50 , (c) 15/72 , (d) 38/63 .

( ) 3. Assume S is the starting symbol, N_T={S, A, B, C}, T={a,b}. What is the language L which is specified by the following grammar.

S => A	S => C
A => aA	A => aB
B => aBb	B => ab
C => Cb	C => Bb

(a) $L=\{a^ib^j \mid i>j\}$ (b) $L=\{a^ib^j \mid i,j>1, i\neq j\}$ (c) $L=\{a^ib^j \mid i,j\geq 1, i\neq j\}$ (d) $L=\{a^ib^j \mid i<j\}$

( ) 4. Let e and v denote the numbers of vertices and edges. A graph G is a planar graph only if (a) $3e \leq v-6$, (b) $e > 3v-6$, (c) $e \leq 3v-6$, (d) $3e>v-6$

( ) 5. Let $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$. Then,
(a) $a_r^{(p)} = -r2^{r+1} + r/2 + 7/4$, (b) $a_r^{(p)} = 2^{r+1} + r/4 + 5/3$, (c) $a_r^{(p)} = -r2^{r+1} + 7r/4 + 1/2$, (d) $a_r^{(p)} = r2^{r+1} + r/4 + 2/3$

( ) 6. What is the generating function of the numeric function, 0*1, 1*2, 2*3, 3*4, ...
(a) $1/(1+z)^2$, (b) $1/(1-z^2)$, (c) $2z/(1-z)^3$, (d) $z^2/(1-z)$

( ) 7. A machine is described by the following state table:

state	input		output
	0	1	
A	B	F	0
B	A	F	0
C	G	A	0
D	H	B	0
E	A	G	0
F	H	C	1
G	A	D	1
H	A	C	1

How many states of the equivalence machine corresponding to the above machine?
(a) 2, (b) 3, (c) 4, (d) 5.

(命題用紙)

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( ) 8. The Tower of Hanoi problem. r circular rings of tapering sizes are slipped onto a peg with the largest ring at the bottom. These rings are to be transferred one at a time onto another peg and there is a third peg available on which rings can be left temporarily. If, during the course of transferring the rings, no ring many never be placed on top of a smaller one, in how many moves can these rings be transferred with their relative positions unchanged?
(a) $3^r + 1$, (b) $2^r + 3^r + 4^r - 1$, (c) $2^r + 3^r - 1$, (d) $2^r - 1$

( ) 9. Let (A, #) be a group and (H, #) be a subgroup of (A, #). Both are defined in the following:

(A, #)

#	x	y
x	x	y
y	y	x

(H, #)

#	x
x	x

How many distinct left cosets of H with respect to A.
(a) 1 (b) 2 (c) 3 (d) 4

( ) 10. Let a_r denote the total graduate students of our Department of Electrical Engineering in the r'th year. Then $a_r - a_{r-1}$ is the incerase in graduate students during the r'th year. If the incerase in graduate students during each year is always five times the increase during the previous year, what are the total graduate students in the r'th year? It is given that $a_0 = 3$ and $a_1 = 7$.
(a) $2 + 5^r$, (b) $15 + r5^r$, (c) $3+2^r$, (d) $6 + 3r + 2^r$.

( ) 11. Let R be a binary relation on the set of all positive integers such that R={(a,b) | b-a is an odd positive integer}. Then, R is
(a) symmetric, (b) antisymmetric, (c) transitive, (d) reflexive.

( ) 12. A binary relation is called a compatible relation if it is
(a) symmetric and transitive, (b) reflexive and symmetric, (c) reflexive and transitive, (d) antisymmetric and reflexive.

( ) 13. Which statement is incorrect:
(a) A monoid is a group. (b) A monoid is a semigroup, (c) A group is a semigroup, (d) A group is a monoid.

( ) 14. If f is an onto function from A to B such that for any a_1 and a_2 in A, $f(a_1 * a_2)=f(a_1)\#f(a_2)$, then f is called a **(a)** homomorphism, **(b)** isomorphism, **(c)** automorphism, **(d)** ring, from (A,*) to (B,#). Here, (A,*) and (B,#) are two algebraic systems.

B. Short answers. The points given are shown at the end of each question

15. Let ({a,b}, *) be semigroup where b*b=a. Show that a*a=a. (10%)

16. A partially ordered set is a lattice if every two elements in the set have a unique ____ and a unique ____. (6%)

17. A Boolean lattice is a ____ and ____ lattice. (8%)

18. An algebraic system (A, +, *) is a ring if (A, +) is a commutative ____, (A, *) is a ____, and the operation * is distributive over the operation +. (6%)