

八十五學年度國立台灣工業技術學院研究所碩士班招生考試

所別：機械工程技術研究所

組別：控制組

科目：線性系統控制

1. Consider a DC motor system which has a transfer function $\frac{\Omega(s)}{V(s)} = \frac{a}{s+a}$, where $\Omega(s)$ and $V(s)$ are the Laplace transform of the motor speed $\omega(t)$ and input voltage $v(t)$ respectively. A Pulse-Width-Modulated (PWM) signal $v(t)$ as shown in Fig. 1a is used as input to the motor. $v(t)$ has period T and duty cycle t_1/T , with $T=t_1+t_2$. In the steady state, the motor speed $\omega(t)$ will have the response as Fig. 1b.

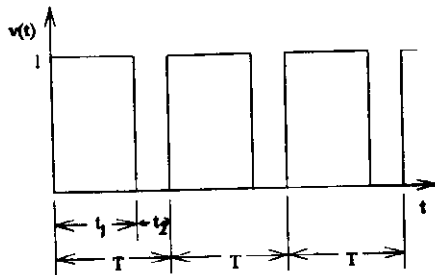


Fig. 1a

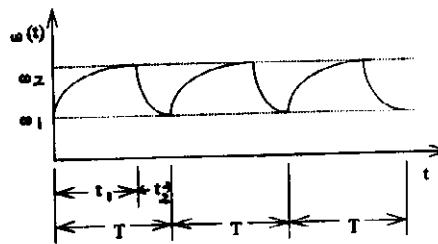


Fig. 1b

Show that in the steady state,

$$\omega_1 = \frac{e^{-a(T-t_1)} - e^{-aT}}{1 - e^{-aT}}, \quad \omega_2 = \frac{1 - e^{-at_1}}{1 - e^{-aT}}$$

2. A Permanent-Magnet (PM) type DC motor has a voltage input $e(t)$ and position output $\theta_m(t)$. The motor has armature inductance L and armature resistance R , with $L \ll R$. Motor torque T_m is proportional to the armature current i_a by a constant K_t , and the back emf e_b generated by the motor is proportional to its rotation speed ω_m by another constant K_b . The armature itself has moment of inertia J_a and damping D_a . The motor is coupled with a combined rotational and translational mechanical system by a spring K_1 as shown in Fig. 2. The rotational moments of inertia (J_i), damping (D_i), spring constants (K_i), gear tooth number (N_i), translational mass M , and pinion radius r are all designated in the figure. The gears are massless. Find the transfer function between the motor speed and the input voltage, $\frac{\theta_m(s)}{E(s)}$.

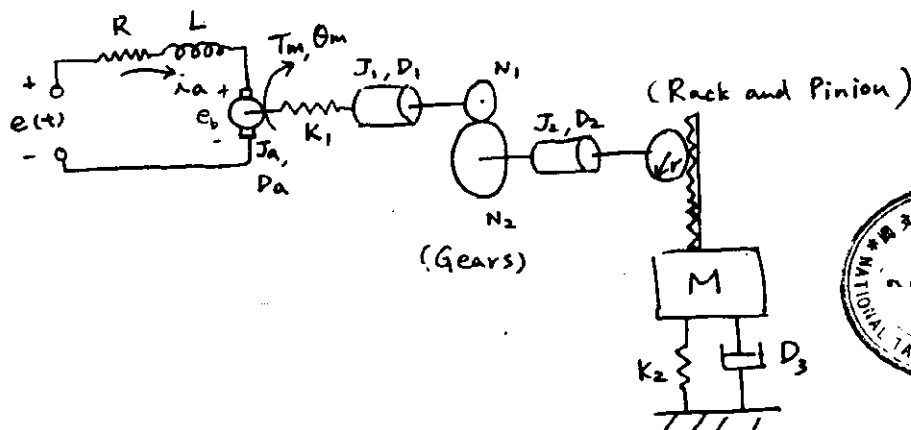


Fig. 2
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3. A control system is shown in Fig. 3.

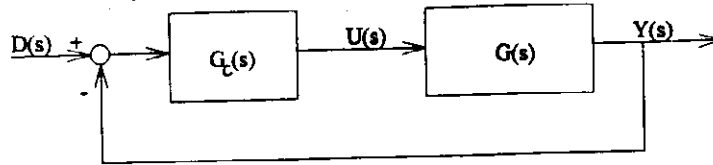


Fig. 3

where the state space representation of G(s) is as follows:

$$\dot{X} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad -2 \quad 1]X$$

- (10%) (a) Find the transfer function $G(s)=Y(s)/U(s)$.
- (15%) (b) Design a Proportional-Derivative (PD) controller $G_c(s)$ so that the closed-loop system has poles at $-0.75+1.5j$ and $-0.75-1.5j$.

4. A feedback control system is shown in Fig. 4.

(25%)

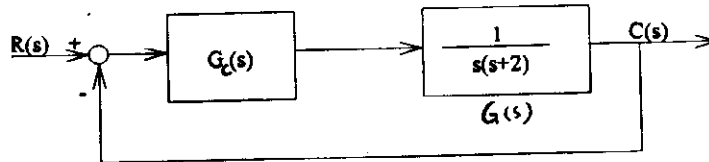
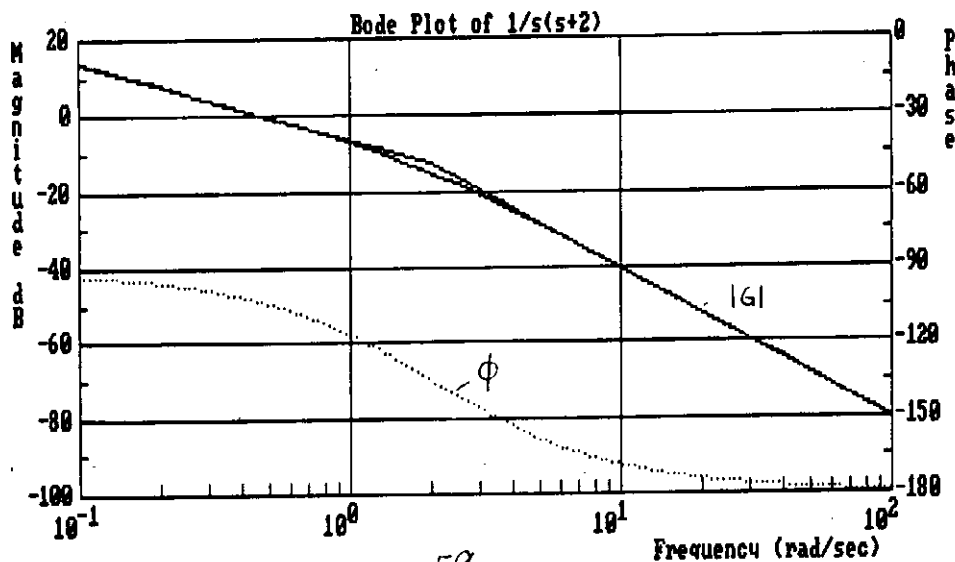


Fig. 3



We would like the system to have steady-state error less than 0.05 for unit ramp input, and phase margin 45 degrees. Design a compensation in the form of $G_c(s) = K \frac{1+as}{1+bs}$ to achieve the specification. Write down your design procedure clearly.



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