

所 別：工程技術研究所
學程別：自動化及控制學程

組別：乙組

科目：作業研究

1. Three professors must be assigned to teach six sections of Operations Research. Each professor must teach two sections and each has ranked each of the six time periods during which Operations Research is taught, as shown in the following table. A ranking of 10 means that the professor wants to teach at that time, and a ranking of 1 means that he or she does not want to teach at that time. Determine an assignment of professors to sections so as to maximize the total satisfaction of the professors. (25%)

	8 A.M.	9 A.M.	10 A.M.	11 A.M.	1 P.M.	2 P.M.
Professor 1	8	7	6	5	7	6
Professor 2	9	9	8	8	4	4
Professor 3	7	6	9	6	9	9

2. Consider the following linear programming problem:

$$\text{Maximize } z = 2x_1 + x_2 + x_3$$

$$\text{subject to } x_1 + x_3 \leq 1$$

$$x_2 + x_3 \leq 2$$

$$x_1 + x_2 \leq 3$$

$$x_1, x_2, x_3 \geq 0$$

It is given that

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

- (a) Show that the basic solution with basic variables x_1 , x_2 and x_3 is optimal. Find the optimal solution. (10%)
- (b) Write down the dual to this problem and find its optimal solution. (10%)
- (c) What is the new optimal solution if the right-hand-side of each constraint is multiplied by a nonnegative constant k ? (5%)

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3. Consider the following nonlinear programming problem:

$$\text{Minimize } f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2)^2 - 3(x_1 + x_2)$$

subject to

$$4x_1 + x_2 \leq 20$$

$$x_1 + 4x_2 \leq 20$$

$$x_1 \geq 0, x_2 \geq 0$$

(1) Obtain the KKT conditions for this problem. (15%)

(2) Derive the optimal solution from the KKT condition if it is known that the optimal solution does not lie on the boundary of the feasible region. (10%)

4. An $M/M/1$ queueing system permits at most K customers in the queue, and customers are blocked and leave the system when the queue is full.

(a) Draw the rate diagram for this system. (5%)

(b) Under what values of $\rho (= \lambda/\mu)$ will this system eventually reach steady-state conditions? Find P_0 and P_n for this system. (10%)

(c) What is the probability that a customer is blocked from the system? (5%)

(d) Find L and W for the case when $\lambda = \mu$. (5%)