

所 別：工程技術研究所
學程別：資訊工程學程

組別：

科目：離散數學

1. Show that $7x^2$ is $O(x^3)$ and x^3 is not $O(7x^2)$. (10%)
2. What are equivalence relations and what are partial ordering relations? (8%) Give an example for each and explain. (8%)
3. The chromatic number of a graph is the least number of colors needed for a coloring of the graph where adjacent vertices have different color. Draw two 6 vertices graph with chromatic number 2 and 3 respectively. (6%)
4. Show that the language $L = \{a^k b^k | k \geq 1\}$ is not a finite state language. (10%)
5. In how many ways can the 26 letters of the alphabet can be permuted so that none of the patterns *car*, *dog*, *pun*, or *byte* occurs? (8%)
6. Let G be a cyclic group. Show that if $|G|$ is infinite, then G is isomorphic to $(\mathbb{Z}, +)$, where $(\mathbb{Z}, +)$ is a group and $\mathbb{Z}$ is a set of integers. (20%)
7. Let H_j be a harmonic number and $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{j}$ for any positive integer j . Show that $\sum_{j=1}^n H_j = (n+1)H_n - n$. (15%)
8. Let $f(1) = 1$ and $f(n) = 2f(\frac{n}{2}) + n - 1$, where $n = 2^k$ and k is a positive integer. Please derive the closed form of $f(n)$. (15%)