

所 別：管理技術研究所
學程別：工業管理學程

組別：

科目：作業研究

1. Consider the following linear programming problem:

$$\begin{aligned} \text{Maximize } z &= 2x_1 + x_2 + x_3 \\ \text{subject to } & x_1 + x_3 \leq 1 \\ & x_2 + x_3 \leq 2 \\ & x_1 + x_2 \leq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

It is given that

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

- Show that the basic solution with basic variables x_1 , x_2 and x_3 is optimal. Find the optimal solution. (10%)
- Write down the dual to this problem and find its optimal solution. (10%)
- What is the new optimal solution if the right-hand-side of each constraint is multiplied by a nonnegative constant k ? (5%)

2. For a computer to work properly, three subsystems of the computer must all function properly. To increase the reliability of the computer, spare units may be added to each subsystem. It costs \$100 to add a spare unit to subsystem 1, \$300 to add a spare unit to subsystem 2, and \$200 to add a spare unit to subsystem 3. As a function of the number of added spare units (a maximum of two spares may be added to each subsystem), the probability that each one will work is given in the following table. Given that \$600 is available for spare units, what is the maximum probability that the computer will work properly? (25%)

No. of spares	Probability that a subsystem works		
	Subsystem 1	Subsystem 2	Subsystem 3
0	0.85	0.60	0.70
1	0.90	0.85	0.90
2	0.95	0.95	0.98

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3. The City Police Department has 5 patrol cars. A patrol car breaks down and requires service once every 30 days. The police department has two repair workers, each of whom takes an average of 3 days to repair a car. Breakdown times and repair times are exponential.
- (a) Determine the average number of police cars in good condition. (10%)
- (b) Find the average down time for a police car that needs repairs. (10%)
- (c) Find the fraction of the time a particular repair worker is idle. (5%)
4. Based on a four-quarter system, the students of a university switch among the various colleges every quarter according to the following probability transition matrix:

		To		
		Engineering	Liberal Arts	Business
From	Engineering	0.5	0.3	0.2
	Liberal Arts	0.1	0.7	0.2
	Business	0.1	0.1	0.8

Assume there are 3000 students in Engineering college, 5000 in Liberal Arts college, and 2000 in Business college at the beginning of the fall quarter.

- (a) Forecast the number of students in each college at the end of the third quarter. (15%)
- (b) Determine the steady-state conditions for the university. (10%)