所　別：管理技術研究所
學程別：資訊管理學程
組別：丙組
科目：作業研究

1.Let X denote a continuous random variable with cumulative distribution function $F(x)$.
(a)(5%) Find the distribution of the random variable $F(X)$.
(b)(5%) When testing $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$, H_0 is rejected if the sample average $\bar{X}$ is greater than some critical value c such that $P\{\bar{X} > c|H_0 \text{ holds}\}$ equals to some prespecified type I error rate α. One way to test the statistical hypothesis is to compute the p-value $= P\{\bar{X} > \bar{x}|H_0 \text{ holds}\}$, where $\bar{x}$ is the realized sample average, and reject H_0 if p-value is less than α. Find the distribution of p-value.

2. Consider the simple linear regression model

$$Y_i = \theta + \beta x_i + \varepsilon_i, \quad i = 1, 2, \ldots, n,$$

where ε_is are independent normally distributed random variables with mean 0 and variance σ^2. Suppose a random sample $\{(x_i, Y_i), i = 1, 2, \ldots, n\}$ is available.
(a)(5%) Find the least squares estimator, $\hat{\theta}$, of θ.
(b)(5%) Find the distribution of $\hat{\theta}$
(c)(5%) Suppose σ^2 is known. Construct a $100(1-\alpha)\%$ confidence interval for θ.

3. *Memoryless* property is an important characteristic of the exponential distribution. Let X be an exponential random variable, then *memoryless* property can be expressed as $P\{X > s + t|X > s\} = P\{X > t\}$; that is, if the life time of a machine follows an exponential distribution then an old machine is not different from a brandnew one as long as it is functioning. Consider a system with three machines connected in parallel. The system functions as long as one machine is working. Suppose three machines start working in the beginning and the life times of three machines are independent exponential random variables with mean μ.
(a)(5%) What is the mean life time of the machine that fails first?
(b)(5%) What is the mean life time of the system?

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4. (15%) Show that if a matrix is positive definite, then the matrix is nonsingular.

5. (15%)
(a) Please explain what is the Jacobi method which is an indirect method to solve the linear system.
(b) Using the Jacobi method to solve a linear system $AX = b$, show that if the matrix A is diagonally dominant, then the convergent solution of X can be obtained for sufficiently large iterations.

6. (12 %) Suppose that a player has \$1 and with each play of the game wins \$1 with probability $p > 0$ or losses \$1 with probability with probability $1 - p$. The game ends when the player either accumulates \$3 or goes broke. Formulate this problem as a Markov chain and identify the classification of its states. Calculate the steady state probabilities for each state if $p = 0.4$.

7. (15 %) Consider the following linear program:

$$\text{Maximize } 15x_1 + 30x_2 + 20x_3$$

s.t.

$$\begin{aligned} x_1 \qquad\quad + x_3 &\le 4 \\ 0.5\,x_1 + 2x_2 + x_3 &\le 3 \\ x_1 + x_2 + 2x_3 &\le 6 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

a. Write down the dual problem.
b. Solve both the primal and the dual problems optimally.
c. What is the dual prices for the three primal constraints?

8. (8 %) Marty's Shop has one barber. Customers arrive at the rate of 3 customer per hour, and haircuts are given at the average rate of 5 per hour. Use the Poisson arrivals and exponential service times model to find Prob{no customer in the system}, L (the expected number of customers in the system), W_q(the expected waiting time in the queue), and Prob(1 customer is receiving haircut and 1 customer is waiting) = ?