

所 別： 電機工程技術研究所  
學 程 別：

組 別： 電力組、控制組

科 目： 工程數學

1. In proof testing of circuit boards, the probability that any particular diode will fail is 0.01. Suppose a circuit board contains 200 diodes.
- How many diodes would you expect to fail, and what is the standard deviation of the number that are expected to fail? (4%)
  - What is the (approximate) probability that at least four diodes will fail on a randomly selected board? (4%)
  - If five boards are shipped to a particular customer, how likely is it that at least four of them will work properly? (A board works properly only if all its diodes work.) (4%)
2. A restaurant serves three fixed-price dinners costing \$7, \$9, and \$10. For a randomly selected couple dining at this restaurant, let  $X$  = the cost of the man's dinner and  $Y$  = the cost of the woman's dinner. The joint probability mass function (pmf) of  $X$  and  $Y$  is given in the following table:

$p(x, y)$		$y$		
		7	9	10
$x$	7	0.05	0.05	0.10
	9	0.05	0.10	0.35
	10	0.00	0.20	0.10

- Compute the marginal pmf's of  $X$  and  $Y$ . (3%)
  - What is the probability that the man's and the woman's dinner cost at most \$9 each? (3%)
  - Are  $X$  and  $Y$  independent? Justify your answer. (3%)
  - What is the expected total cost of the dinner for the two people? (3%)
3. Assuming that the stated inverses exist, prove the following equalities.
- $(C^{-1} + D^{-1})^{-1} = C(C + D)^{-1}D$ . (4%)
  - $(I + CD)^{-1}C = C(I + DC)^{-1}$ . (4%)

4. Find  $A^n$  if  $n$  is a positive integer and

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}. \quad (10\%)$$

5. Let  $B = \{v_1, v_2, v_3, v_4\}$  be a basis for a vector space  $V$  and  $T: V \rightarrow V$  the linear operator for which

$$\begin{aligned} T(v_1) &= v_1 + v_2 + v_3 + 3v_4, \\ T(v_2) &= v_1 - v_2 + 2v_3 + 2v_4, \\ T(v_3) &= 2v_1 - 4v_2 + 5v_3 + 3v_4, \\ \text{and } T(v_4) &= -2v_1 + 6v_2 - 6v_3 - 2v_4 \end{aligned}$$

- Find the rank and nullity of  $T$ . (4%)
- Determine whether  $T$  is one-to-one. (4%)

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6. Find the general solution to

$$y'' - \left(\frac{1}{x}\right)y' + \left(\frac{1}{x^2}\right)y = x^2 + 1, \text{ for } x > 0 \quad (10\%)$$

7. Find the general solution to

$$y''' - y'' - 8y' + 12y = 7e^{2x} \quad (10\%)$$

8. Find the first five nonzero terms of the Maclaurin series of the general solution of the following differential equation. Also find the recurrence relation for the coefficients in the series solution.

$$2y'' - 4xy' + 8x^2y = 0 \quad (10\%)$$

9. Evaluate

$$\int_0^{2\pi} \frac{\cos \theta}{1 + \frac{1}{4}\cos \theta} d\theta \quad (10\%)$$

10. Map  $|z| < 2$  onto the domain  $D^*$ :  $u + v > 0$  in the  $w$ -plane. (10%)